STARTPAGE

HUMAN RESOURCES AND MOBILITY (HRM) ACTIVITY

MARIE CURIE ACTIONS

Research Training Networks (RTNs)

PART B

PHD

TABLE OF CONTENTS

List of teams

Participant No.	Location	Scientist-in-charge
Team 1	Marne-la-Vallée	A. Pajor
Team 2	London	K. Ball
Team 3	Kiel	H. König
Team 4	Budapest	M. Simonovits
Team 5	Heraklion	A. Giannopoulos
Team 6	Warsaw	P. Mankiewicz
Team 7	Zaragoza	J. Bastero
Team 8	Florence	A. Colesanti
Team 9	Vienna	P. Gruber
Team 10	Toulouse	M. Ledoux
Team 11	Freiburg	R. Schneider
Team 12	Tel Aviv	V. Milman
Team 13	Kharkov	L. Pastur

B1 SCIENTIFIC QUALITY OF THE PROJECT

B1.1. Research topic

Over the last 20 years there has been a dramatic growth in our understanding of phenomena that occur in high-dimensional systems: those whose characteristic behaviour appears as the number of variables grows to infinity. Such systems occur in a wide variety of branches of mathematics and adjacent sciences, in particular, physics and computing. It is clear that the ability to describe basic features of such systems mathematically, will be increasingly important in the life sciences as well, during the next decade.

Numerical simulations and theoretical analyses have revealed two characteristic features of high-dimensional phenomena which can be described as "unexpected uniformity" and "sharp discontinuity". Historically, these characteristics appeared first in random settings, especially in probability (various forms of the law of large numbers and other limiting theorems), and in statistical physics, where systems of particles, described by the Gibbs distribution, exhibit phase transitions: sudden qualitative changes in behaviour as a result of small changes in parameters. It has become increasingly clear that what underlies these effects is not randomness itself but rather a broader concept of high-dimensionality. High-dimensional systems occurring naturally exhibit random-like behaviour.

The recognition of this occurred almost simultaneously in several fields: high-dimensional geometry and combinatorics as well as probability and statistical physics themselves. Until recently, these branches of mathematics were considered quite separate; and this impeded the exchange of ideas and methods even though the problems and methods were inherently related.

The goal of this project is to bring together researchers in the areas of geometry, analysis, combinatorics, mathematical physics and probability in which these phenomena have emerged and have been studied, and thereby to provide young scientists with a considerably wider understanding of the unity of the respective mathematical ideas. Among the most senior scientists in the programme are many of the architects of the last 20 years' developments; for example, **Alon, Gowers, Gromov, Lovász, Milman, Pastur, Pisier**, **Talagrand** and **Tsirelson**.

While the achievements of the last 20 years have been impressive, there are serious reasons to believe that we are poised to provide a strikingly clearer picture of how to handle high-dimensional phenomena in the areas in question: for example, **Talagrand** has just solved the long-standing problem on the form of the free energy in the Sherrington-Kirkpatrick model of spin glasses. His work depends heavily on deviation estimates originally discovered in probabilistic and geometric settings. It should be noted that this direction of mathematical statistical physics was initiated by works of participants of the Ukrainian team in the early 90's.

Although the members of the network come originally from different mathematical disciplines and their subjects may seem somewhat fragmented, their works are linked by a common experience of high-dimensional phenomena and more importantly by a commonality of methods which have been found to be applicable in all these areas. A fundamental example is that of concentration principles, which underly a majority of themes of this program, and which guarantee that within apparently diverse families, almost all members look similar: that there is a universal "type" which is mimicked by almost all members. Examples of this are the self-averaging properties in random matrix theory and statistical physics of disordered spin systems, the concentration phenomenon in geometry as exemplified by Dvoretzky's theorem and later work in the same direction, and the recent appearance of limit shapes for random bodies. Quantitative measures of concentration are essential for randomised computation: they are, in effect, the definition of geometric expanders and recently they were found to imply strong quantitative results concerning entropy growth in the central limit theorem.

B1.2. Project objectives

This project arises from the confluence of several branches of mathematics. The objectives of the project are supplied by these different branches but should be understood as part of an overall goal: to understand the appearance of "unexpected uniformity" and "sharp discontinuity" in high-dimensional structures. The six main areas of the programme are as follows:

- Asymptotic Geometric Analysis
- Isometric Convex Geometry
- Asymptotic Combinatorics
- Randomised Computation and Complexity
- High-dimensional Phenomena in Mathematical Physics
- Isoperimetric Principles in Geometry and Probability

Researchers in the area of **Asymptotic Geometric Analysis** will continue the search for phase transitions and threshold behaviour in high-dimensional normed spaces: in particular, in the study of the linear structure of such spaces. A number of remarkable results in this direction were discovered by the group in recent years but they look to be just the "tip of an iceberg". For the first time there is now an opportunity to attack the so-called "slicing problem" (which goes back to Bourgain in the 80's), one of the most notorious problems in Asymptotic Convexity. (The problem has many equivalent formulations: one such is that the geometric average of the principal moments of inertia of a convex body of volume 1, is bounded independently of the dimension and the body). Progress on this problem by **Milman**, his student Klartag and Bourgain has opened a new avenue in this direction. A related and extremely intriguing issue which is attracting significant attention concerns the "Central Limit Properties of Convex Bodies". One form of the question is whether every isotropic convex body of high enough dimension has a direction in which the projection of its volume is close to a Gaussian distribution. There is an obvious link with classical probability here but the real point is to see the classical central limit theorem as a reflection of underlying geometric principles. The methods used to attack many of the earlier problems relied heavily on entropy and covering properties. Recently a major old conjecture on this subject, the Duality of Entropy Conjecture, started to "crack" and we hope for a major breakthrough in this direction.

Over the last decade, research in **Isometric Convex Geometry** has begun to make connections with more and more areas of mathematics. The relations of convexity to functional analysis, probability and combinatorics are emphasized by the project and will be promoted further by it. One of the main objectives of the project within convex geometry, is the continued study of the approximation of convex bodies by polytopes. The topic has been enormously influenced by random methods as shown by the recent discoveries of several members of the net that the rates of random approximation and best approximation are essentially the same. The objectives here are the extension to non-smooth convex bodies of the asymptotically optimal formulas for the degree of approximation which are known for smooth bodies, the expansion of asymptotic formulas in the case of best approximation and the construction of efficient algorithms for performing approximation.

Classical geometric inequalities such as the Brunn-Minkowski inequality have strong links with another main subject of the program: **Isoperimetric Principles in Geometry and Probability**. The most challenging questions in this direction concern stability results for geometric inequalities, which lead to better understanding of these inequalities, sharper versions and dramatic new applications.

The **Asymptotic Combinatorics** group has several long term goals in combinatorics and theoretical computer science. One of them is to understand better the asymptotic behavior of graph theoretic parameters in various models of random graphs, in particular, parameters related to the spectrum, such as eigenvalues, the **Lovász** theta-function and vector chromatic number. The study of these parameters cuts across several mathematical disciplines such as spectral theory, the theory of random matrices and the theory of algorithmic complexity, where these parameters are widely used. Several recent results obtained by the group include the determination of the asymptotic value of the choice number in random graphs (**Krivelevich**) and random bipartite graphs (**Alon** and **Krivelevich**) and the concentration of eigenvalues of random symmetric matrices and its algorithmic applications (**Alon, Krivelevich** and Vu). It is to be expected that there will be further important progress in these areas.

Pseudo-random graphs can be informally defined as graphs whose properties resemble those of truly random graphs of the same density. The group plans to study pseudo-random graphs, and to find new constructions of them. Again there has been important recent work in this area: new constructions of geometric expanders have applications in computer simulation as well as providing concrete models of randomness.

The third thread of asymptotic combinatorics is Ramsey Theory. There are some exciting new bounds on Ramsey numbers recently provided by members of the group while **Gowers** has identified the possibility of finding very much more effective methods in the direction of **Szemerédi's** Theorem and its higher-dimensional generalisations.

The area of **Randomised Computation and Complexity** covers applications of methods from probability theory and convex geometry to the design and analysis of algorithms. A benchmark problem has been that of computing the volume of high-dimensional convex bodies: there has been enormous progress, but the task is still not finished. Recent advances (many by project members) have opened up new possibilities, such as the use of simulated annealing techniques and the applications of more sophisticated expansion measures. Other important problems concern the computation of geometric invariants such as diameter, center of gravity and inertial ellipsoid that capture more detailed information than the volume.

A difficult and largely open area is to obtain lower bounds on the complexity of (randomised) volume computation and other algorithmic tasks. At this time, only essentially trivial lower bounds are known, and any progress in the direction (for example, a nonlinear lower bound on the minimum number of oracle calls that allow a randomized approximation of the volume) would be very significant.

The **Mathematical Physics** topics within the project fall into three main areas. Universality is a crucial concept in the theory of eigenvalue distribution of large random matrices, and refers to the appearance of common patterns in asymptotic distribution of eigenvalues, that comprise a system of strongly dependent random variables. The group plans to find new approaches to the universality of the local regime (for both the bulk of the spectrum and its edges), valid under less stringent conditions and applicable to a wider class of random matrix ensembles (e.g. orthogonally invariant ones) and new limiting probability laws for linear eigenvalue statistics, that will replace the central limit theorem in the case where the support of the limiting eigenvalue counting function consists of several intervals. The work will involve new techniques based on the analysis of systems of functional equations, asymptotic study of new classes of orthogonal polynomials, abstract Markov operators and related techniques of spectral, logarithmic, and classical Sobolev inequalities.

Within the statistical mechanics of disordered systems one of the most challenging problems is to obtain an explicit form of the most relevant physical quantities (such as free energy and correlation functions) for the principal mean field models. Advances in this direction will follow from further development of the rigorous cavity method, proposed and successfully studied by the Kharkov and Paris teams, combined with certain convexity considerations, that proved to be efficient in recent progress in the field due to Guerra, **Talagrand** and **Shcherbina** and Tirozzi. Some new models in theoretical neurobiology, known as the "integrate and fire" mean field models will be studied by means of the theory of stochastic differential equations and Markov processes.

Studies of spectral and evolution properties of PDE's will be based first of all on the recent considerable progress in the analysis of convexity properties of eigenvalues for boundary value problems defined by elliptic PDE's. In particular, a new proof providing equality conditions, of the Brunn-Minkowski inequality for the first eigenvalue of the Laplacian, was given by members of the Italian team. One of the main tasks will be to find *natural assumptions* on a variational set functional, which ensure that it obeys a Brunn-Minkowski type inequality and unifying results that imply as special cases the existing inequalities. It is also planned to study geometric and analytic properties of global attractors for infinite dimensional dissipative dynamical systems generated by initial boundary-value problems for nonlinear partial differential equations. The focus will be on mixed systems, important in the physics of multi-component and multi-layer structures and in climate and weather modelling, and on the cases where global attractors have no regular structure. It is also planned to find new finite-dimensional approximate inertial manifolds for deterministic and stochastic retarded equations, where infinitely many additional degrees of freedom appear in comparison with the non-retarded case.

In the area of **Isoperimetric principles** it will be important to investigate further the interplay between geometric inequalities, mass transportation and probabilistic and semigroup tools. A new avenue in this direction concerns the (Riemannian) geometry of the space of probability distributions equipped with the Wasserstein distance: it is becoming clearer that there is a connection between the curvature of the space and the displacement convexity of entropy functionals. It is to be expected that the study of convergence to equilibrium of fast diffusions, in the entropy or Wasserstein distances, via functional transportation and Sobolev inequalities will be a fruitful source of interaction with the PDE community. Thanks to the work of project members in the last three years, it has become possible for the first time to attempt a serious study of entropy growth for discrete semi-groups: something which was previously only possible for the extremely restricted class of continuous semi-groups.

B1.3. Scientific originality

The most original feature of this project is that it brings together mathematicians from a variety of disciplines who have witnessed parallel developments within their fields, who share a broad range of methods, and who can benefit by adapting techniques which are not yet known to all disciplines. Within each area of the project there have been startling and highly original results produced in the last 20 years and the best guarantor of continued originality is the outstanding quality of the personnel involved, including several students and more experienced researchers who are already beginning to make their names. Even just to outline the state of the art in all the areas of expertise of the researchers in this project would be impossible in the space provided: below is a sample of recent discoveries and directions in which we expect to make progress.

In the field of asymptotic convexity, the most startling achievement has been the recognition that high-dimensional convex bodies exhibit an unexpectedly uniform structure in terms of their lower dimensional sections and projections. **Milman**'s Q/S theorem, which states that every n-dimensional normed space has an $n/2$-dimensional quotient of a subspace which is essentially Euclidean space is the clearest example. Recently it was found that the level of subspace at which uniformity sets in can be described accurately in terms of a single invariant of the original space. An issue which is closely related in spirit, but is still poorly understood, is the so-called central limit problem for convex bodies: is it true that every convex body has marginal distributions that are almost all essentially Gaussian? A refinement of work by Sudakov and Diaconis-Freedman reduces the problem to a special case of the problem of determining the dimensional dependence of the spectral gap for the Neumann Laplacian on an isotropic convex body. There is a good chance that progress can be made in improving these estimates, at least for the most interesting cases. A student in the Greek team has recently clarified the issues here considerably.

One of the most striking achievements of classical convexity in recent years has been the almost complete understanding of the asymptotic rate of approximation of convex bodies by polytopes and the associated discovery of a limit shape for random bodies, by **Bárány**, **Gruber**, **Schneider**, **Schütt** and others. The asymptotic dependence of the approximation on the number of faces or vertices of the polytope has been exactly described in terms of an invariant of the body, the affine surface area. The major problem here is to refine the methods so as to provide estimates for fixed numbers of faces. This problem is closely related to the slicing problem mentioned above, on which there is a very good chance of progress.

There is a strong chance that extremal graph theory will develop considerably in the next few years. There has been recent progress in understanding when are the extremal structures random-like, or of geometric type and when are the extremal structures describable in very simple ways. Uniform distribution methods have been applied in problems connected to the solution of the Banach-Tarski paradox (**Laczkovich**). One of the most important tools in the field, the regularity lemma, has now been extended to hypergraphs in several cases by Frankl and Rödl and has several new versions, (the blow-up lemma, the counting lemma and the sparse regularity lemma).

Within the parts of mathematical physics covered by this proposal are random matrix theory, the analysis of mean field models of disordered spin systems, and variational and evolution approaches to PDE's . The theory of random matrices has been spectacularly successful in the last few years. The main aim is to describe the statistical properties of eigenvalues of matrices whose entries satisfy probabilistic hypotheses. The theory has applications to combinatorics (Ulam's problem on random permutations, the spectral description of random graphs) and probability theory (first-passage percolation) as well as physics, and has inspired a striking paradigm for the distribution of zeros of the Riemann zeta function following work of Montgomery, Odlyzko, Sarnak and others. The overall distribution of eigenvalues (the global regime) is by now well-understood for the most important matrix ensembles: the distribution of individual eigenvalues and their gaps (the local regime) is only just becoming tractable. Efficient bounds for the norm of random matrices were recently found within the framework of the concentration phenomenon by **Szarek, Alon, Krivelevich** and Vu, **Blower**, and

Latała. The typical size of the second largest eigenvalue of a random matrix was recently described by **Füredi** and Komlós. It is here that we expect to make significant advances, proving the universality of the eigenvalue distribution in the local regime under minimal assumptions and finding new forms of limiting distributions of properly normalized linear statistics of eigenvalues. We are going to develop new approaches and concepts, based on concentration methods of asymptotic geometry, new results on asymptotics of certain orthogonal polynomials, certain functional equations, and on splitting the scales, in particular, the appearance of the new adiabatic scale in the analysis of the spectrum edges. Diluted matrix models are very popular now. There are many physically motivated conjectures on their limiting behavior. However there are only few mathematical results on the topic. We are going to contribute to the field by studying, by means of combinatorial techniques, the limiting eigenvalue counting measure and its fluctuations for the discrete Laplacian on random graphs.

There is a vast literature on spin systems which analyses phase transitions, replica symmetry and its breaking and the self-averaging property of free energy, both rigorously and using sophisticated heuristics. Recent breakthroughs include **Talagrand**'s remarkable analysis of the Sherrington-Kirkpatrick model and the solution of the Gardner problem (Kharkov team), by a combination of statistical mechanics and probability tools with Brunn-Minkowski principles inspired by classical convexity. Rigorous results of this kind have been a long time coming: it is reasonable to expect that these breakthroughs are a prelude to an enormously improved understanding of spin systems.

Works of Ladyzhenskaya, Foias, Temam, Constantin, Vishik, and others in the last two decades have shown that the long time behavior of basic dissipative evolution equations of mathematical physics is characterized by global attractors. Their existence and finite dimensionality were proved for a wide class of problems while the structure of attractors was completely described only for certain types of PDE's. In this context approaches that study analytic and geometric properties of attractors, in particular, minimum sets of natural parameters that determine attractors uniquely, look extremely promising. Recent particular results by Foias, Robinson, **Chueshov** indicate strongly that this direction provides a route for attacking more general problems. It is believed that the combination of this with ideas and techniques of asymptotic analysis will boost constructive analysis of mixed and composite systems of evolution equations, in particular, a finite dimensional description of the long time dynamics of a number of infinite dimensional systems and their random perturbations.

B1.4. Research method

In mathematics it is impossible to predict in advance what methods will ultimately succeed in solving a problem or generating insight into the structure of a mathematical object. Indeed, for most problems, it is necessary to find some methods specific to the problem as well as to invoke existing ideas. However, in the context of this programme there are a range of methods that are frequently found to be useful in combination with problem-specific arguments. Moreover there is an overarching theme which serves to frame many specific problems. This theme has already been mentioned under the name, concentration phenomenon: the tendency of high-dimensional systems to congregate around typical forms. One of the best examples of this comes from asymptotic convex geometry. On the face of it, convex bodies in high dimensions exhibit a chaotic diversity, increasing rapidly with the dimension. In fact, it has been found that in many important respects, they exhibit less and less diversity as the dimension increases; both in the sense that all bodies share important common features and in the sense that most bodies have extremely precise similarities. Similar situations occur in mathematical physics where the phenomenon is known as self-averaging.

Concerning the more technical methods that are common to all or almost all of the areas involved in this proposal, there are certainly too many to describe in detail: the following is a representative sample.

Symmetrisation The term symmetrisation covers a range of techniques: Steiner and Minkowski symmetrisation, combinatorial compression, semigroup methods, isomorphic symmetrisation, transportation of measure and Gaussian comparison principles. In each case, the object of interest is transformed, by a sequence of small steps, into a highly regular or symmetric object whose properties can be understood directly. The problem is then to understand what happens at each small step (and possibly, how many steps are needed). In the case of measure transportation, the transformation is completed in one step, but the transformation has a special structure that can be analysed locally.

Several of these methods have been enormously refined by members of the network during the last 15 years. In the case of Steiner symmetrisation, Hadwiger showed in the 50's that at most n^n symmetrisations are needed to transform a convex body into an approximate Euclidean ball. Bourgain, Lindenstrauss and **Milman** showed that $n \log n$ would suffice and recently, the logarithmic factor was removed in a striking article by a student from the Israeli team. For evolution of random variables along convolution semigroups, **Ball** and **Barthe**, together with two students, recently found a new approach to tracking entropy growth along the evolution, allowing the first quantitative estimates for the speed of convergence to the Gaussian limit.

Random selection Many parts of the picture that underlies this project arose in the following way. It was initially guessed that within each apparently chaotic high-dimensional structure of a particular type, could be found highly symmetric or regular regions, of slightly smaller dimension or size. The existence of such regions was demonstrated by random selection: it was shown that a randomly chosen smaller region would (with high probability) possess the desired regularity. But the solution to the existence problem, changed the picture. Except in unusual situations, random selection does not locate rare objects: a randomly chosen object is (by definition) typical. Thus, far from being oases of regularity, the well-behaved regions almost fill the original structure, being separated only by narrow regions of chaotic behaviour. This in turn makes it possible to recover global or large-scale information about the original structure, by rebuilding it from the widely appearing regular parts.

A perfect example of this is the **Szemerédi** regularity lemma, which describes in a precise way, a strong sense in which every network can be modelled by a random one. This lemma was originally proved as a step towards **Szemerédi**'s solution of the Erdös-Turán-Roth problem on the existence of long arithmetic progressions in dense sequences, but is now widely used for understanding graphs and their matrix relaxations. Other examples are the self-averaging and the cavity method in the statistical mechanics of disordered systems and universality in random matrix theory.

Deviation or isoperimetric principles: spectral gap estimates Deviation principles state, in one form or another, that a function which does not experience large jumps over small ranges cannot vary much over large ranges. There is a crude sense in which this is obvious: the total variation cannot be more than the sum of variations over small steps. But in many situations, enormously stronger statements have been found, which show that not only are the individual steps small, but that in addition, they almost entirely cancel one another, so that the typical variation over large distances is far smaller than crude estimates would imply.

The classical central limit theorem is just such a statement, in the special case of linear combinations of independent random variables. The work of **Talagrand** and of **Bakry** and Emery in the early 90's, and of **Ledoux** and his collaborators in the late 90's demonstrated similar estimates for very general product measures, in which the linearity has completely disappeared. In the work of **Gromov** on deviation inequalities on Riemannian manifolds, not only is there no linearity but also the product structure is replaced by a precise geometric structure. One of the main problems for several groups in this project is to obtain (less precise but similar) inequalities in situations where the geometric and/or analytic hypotheses are much coarser.

In another direction, probabilists have studied deviation principles for chaos, in which linearity is replaced by multi-linearity. **Kwapień**, **Latała** and **Oleszkiewicz** from the Polish team are experts in this direction.

Deviation estimates are closely related to Poincaré or spectral gap inequalities. An estimate for the spectral gap of the Neumann Laplacian with respect to a density guarantees a deviation estimate for functions integrated against this density. Within combinatorics, geometric expanders are characterised by the spectral gap for the transition kernel of the corresponding random walk. The existence of spectral gaps for many nice densities is immediate from compactness arguments and is widely used in the spectral theory of PDE's and statistical mechanics but the dependence of the spectral gap upon dimension, for densities satisfying natural geometric conditions, is far from being understood. This illustrates clearly the sense in which the high-dimensional or asymptotic focus of this project requires a refined analysis of issues that appear qualitatively in other areas of mathematics.

Ramsey methods: exhaustion arguments These are, in a sense, a complement to random selection. Ramsey theory aims to demonstrate the existence of rare regions of regularity, rather than identify typical ones. In the simplest situations, this is achieved by an incremental process in which the region of regularity is enlarged slightly, as the original object is expanded by a much larger amount. However, in his recent work **Gowers** used a mixed approach which almost certainly owed something to his background in asymptotic geometry. By means of methods of harmonic analysis (in the spirit of the Hardy-Littlewood circle method) he established a dichotomy in which a dense sequence either contains many arithmetic progressions (as if they are typical) or the sequence possesses some additional density to which an exhaustion can be applied.

B1.5. Work plan

For the detailed work plan it is appropriate to divide the project into the 6 main areas that it encompasses. The main tasks of researchers in each area are detailed together with some milestones.

B1.5.1 Asymptotic Geometric Analysis: Teams 1, 2, 3, 5, 6, 7, 8, 11, 12

MAIN TASKS.

- Solve the "slicing problem" concerning the volumes of sections of convex bodies.
- Solve the Duality of Entropy Problem as generally as possible.
- Investigate further the phase transitions and thresholds in Asymptotic Convexity.
- Understand the central limit properties of isotropic convex bodies.
- Study different symmetrisation procedures in particular the number of steps needed to achieve almost isometric symmetrisation.

MILESTONES.

- Estimates of the influence of symmetrisations on the decay of isotropic constants.
- Solution of the Duality Problem for some specified spaces.
- New examples of phase transitions and thresholds in Asymptotic Convexity.
- Estimates of the number of Minkowski-Blaschke symmetrisations needed to approximate the Euclidean ball with a given precision.

1.5.2 Isometric convex geometry: Teams 1, 2, 3, 4, 5, 7, 8, 9, 11, 12

MAIN TASKS.

- Study random and optimal approximation of convex bodies by polytopes.
- Analyse the asymptotic behaviour of random geometric configurations using methods from ergodic theory, Poisson approximation and the limit theory of (mixing) dependent random variables.
- Develop a unified approach to inequalities of Brunn-Minkowski type for variational functionals.
- Pursue stability estimates for classical inequalities and geometric problems.
- Characterise valuations with interesting geometric properties.

MILESTONES.

- Extension of results on asymptotic approximation currently known under smoothness assumptions to general convex bodies and development of higher order asymptotics.
- Large deviation refinements of limit theorems for random configurations.
- Proof of an analogue of the **Bárány-Buchta** threshold phenomenon for normally distributed points.

1.5.3 Asymptotic combinatorics: Teams 2, 4, 12, 13

MAIN TASKS.

- Study asymptotic behaviour of random graphs and other combinatorial structures: in particular the study of spectrum-related parameters in random graphs and their algorithmic applications.
- Use pseudo-random graphs to capture deterministically, the properties of random graphs (especially for use in algorithms).
- Improve estimates for Turán and Ramsey numbers.

MILESTONES.

- More precise determination of the asymptotic behaviour of graph eigenvalues and related parameters in models of random graphs.
- Description of new properties ensuring pseudo-randomness, their comparison and equivalence.
- Description of new explicit constructions of pseudo-random graphs.

1.5.4 Randomized computation and complexity: Teams 2, 4, 11, 12

MAIN TASKS.

- Find improved volume algorithms, in particular, find ways to eliminate the sensitive dependence on the starting point of the random walk.
- Analyse the dependence on the step size, find transference principles between walks with different step sizes.
- Find relationships between the convergence of a geometric Markov chain to stationarity and the convergence of various computable parameters of the distribution after a given number of steps.
- Find lower bounds on the complexity of volume computation.
- Find applications of randomized methods to the computation of geometric quantities other than the volume.

MILESTONES.

- A rigorous connection between the mixing time of a continuous random walk (Brownian motion) and a discrete step random walk in a convex body.
- Improved algorithms for sampling from a convex body.
- Lower bounds on the complexity of sampling, integration, or volume computation.

1.5.5 High-dimensional phenomena in mathematical physics: Teams 1, 2, 4, 6, 8, 12, 13

MAIN TASKS.

- Develop approaches to the analysis of eigenvalues of random matrices and zeros of random analytic functions, in particular, in the global regime (self-averaging), the intermediate regime (limiting theorems), and local regime (universality).
- Describe in explicit form the limiting behavior of basic models of spin glasses and neural networks (with emphasis on phase transitions).
- Develop a general approach to the study of smoothness properties of global attractors for infinite dimensional dissipative dynamical systems.

MILESTONES.

- Derivation of a functional equation for the limiting reproducing kernel of unitary invariant matrix ensembles in the local regime.
- Concentration type estimates and transportation tightness for counting measures and for extreme values of various classes of random matrices and random analytic functions.
- A proof of a Brunn-Minkowski inequality for the first eigenvalue of the p-Laplace operator and for functionals related to Hessian operators.
- Estimates of the free energy and correlators for the Sherrington-Kirkpatrick model and for the new "integrate and fire" model of neural networks.
- Establishment of regularity properties for the global attractors for plate and wave equations with nonlinear viscous and/or thermal damping.

1.5.6 Isoperimetric principles in geometry and probability: Teams 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

MAIN TASKS.

- Develop a functional or mass transportation approach to the finite dimensional Levy-Gromov comparison theorem.
- Apply mass transportation methods to the Hardy-Littlewood-Sobolev inequalities.
- Find probabilistic representation formulas in the context of the Brunn-Minkowski and Ehrhard inequalities.
- Examine the use of functional inequalities in random matrix theory.

- Extend recent results on entropy growth to random variables satisfying classical probabilistic hypotheses.

MILESTONES.

- Simplified and unified proofs of Hardy-Littlewood-Sobolev inequalities.
- New functional methods for obtaining eigenvalue distributions of random matrices.
- Quantitative estimates for entropy growth of random variables satisfying moment conditions.

The involvement of the teams in each task is illustrated by the following table:

	Teams involved												
Task	1	2	3	4	5	6	7	8	9	10	11	12	13
1	X	X	X		X	X	X	X			X	X	
2	X	X	X	X	X		X	X	X		X	X	
3		X		X								X	X
4		X		X							X	X	
5	X	X		X		X		X				X	X
6	X	X	X	X	X	X	X	X	X	X	X	X	

B2 TRAINING AND/OR TRANSFER OF KNOWLEDGE ACTIVITIES

B2.1. Content and quality of the training and transfer of knowledge programme

Training in mathematics has three principal objectives:

1. To acquaint young researchers with the state of the art in their specialized field, and to train them to identify problems and tackle them.
2. To provide researchers with a broad background of ideas and tools from neighbouring areas of mathematics and science.
3. To introduce young researchers to mathematical and scientific culture in general, including the professional culture with its emphasis upon cooperation and respect for the work of others.

The third of these is usually taught by example, especially the example of the supervisor. However, it depends also upon the prevailing attitudes within a discipline. Several of the fields represented by this programme are famous for the open atmosphere in which research is conducted and the high degree of collaboration. The two recent research programs mentioned below were excellent examples of this.

The first and especially the second objective are the ones that will most directly benefit from the proposed network. Students initially learn from monographs and textbooks and from recent research articles. There are several clearly written introductions to different topics within the program, by **Alon**/Spencer, **Ball**, **Bollobás**, **Ledoux**, **Ledoux/Talagrand**, **Milman/Schechtman**, **Pastur**, **Pisier**, **Schneider** and **Talagrand**. In addition there will be a proceedings of the ICMS educational conference referred to below. Finally, the team in Israel publishes the GAFA seminar notes every couple of years: a widely-respected series that reports on and analyses the main developments in asymptotic geometry and related areas. It frequently includes educational expository articles as well as research articles. Younger researchers will be strongly encouraged to read these expositions.

However, the most important ways in which knowledge is disseminated and researchers are trained is by direct contact with senior mathematicians and through lectures and lecture series at workshops and educational conferences. The visits of postgraduate and postdoctoral researchers to other institutions within the network will be the most important respect in which the complementary expertise of different groups within the network will have training and transfer of knowledge benefits. For example, the Hungarian team has specifically identified the fact that while the Budapest school is world-renowned for its work in computational complexity and algorithms, and combinatorics in general, it has recently been weaker in the field of geometric functional analysis. Thus it can enormously contribute to and benefit from the exchange of young personnel. Similarly, the Austrian and Italian groups have a strong tradition in the classical Brunn-Minkowski theory but have few personnel with probabilistic backgrounds. Even the most diverse groups, the French and British ones, each have obvious areas in which they could gain from contact with others. The fact that the programme encompasses a wide range of mathematical areas makes networking especially important: the success of recent programs covering this wide range shows that the different groups are able to communicate effectively with one another.

The team has been heavily involved in (or entirely responsible for) several extremely successful programs with strong educational content. **Ball** and **Milman** were the principal organisers of a 6-month programme at the MSRI in Berkeley in 1996 which included an introductory workshop aimed at students and members of under-represented groups. In 2003, **Carbery** and **Gowers** from the British team and **Milman** from the Israeli team (along with two mathematicians from the US) organised a two-week instructional conference under the auspices of the ICMS. This involved lecture series on harmonic analysis, combinatorics, probability and geometry given by leading figures in their fields, two-thirds of whom are participants in this network (the other third being from the US). The conference was extremely well attended by students and postdoctoral researchers from around Europe, most of whom are members of the proposed network. In 2003 **Milman**, **Schechtman** and three Canadian mathematicians organised a 6-week programme at PIMS in Canada. This programme brought together researchers in all the main areas of the project and the high quality of the lectures and dedication of the senior personnel made it possible for everyone (especially students and postdocs) to grasp all the material.

The network intends to expand considerably this tradition of broadly based and educational meetings. The network will organise one or more shorter workshops each year, on specific topics within the programme, where the emphasis will be on training and transfer of knowledge. Workshops already planned are

- Toulouse (January 2005): "Random matrices and probability"
- Florence (Spring 2005): "Geometric inequalities"
- Budapest (January 2007): "Asymptotic combinatorics: methods and applications"

It is intended that there will be workshops organised by the Freiburg node (2006) and the Tel Aviv node (2008). There will also be a research conference each year with a strong educational content. Most importantly, the network will organise a 1-month summer or winter school each year, to which all younger personnel will be encouraged to go. At each school, a selection of senior scientists from the programme will give expository lecture series. The first two are already arranged. The intended schedule is as follows:

- Vienna, Schrödinger Institute (July 2005), "High-dimensional geometry and convexity".
- Paris, IHP (Spring 2006), "High-dimensional phenomena" (see below).
- Warsaw (Summer 2007), "Probabilistic methods in geometry".
- Cambridge (Summer 2008), "High-dimensional phenomena".

The network will be able to benefit from the trimester programme in high-dimensional phenomena scheduled to take place at the Centre Emile Borel in Spring 2006, organised by Bourgain, **Milman**, **Pajor**, **Pastur** and **Pisier**. This programme will include a regular seminar series presenting recent discoveries, several long and short lecture series by invited speakers including **Alon**, **Lovász** and **Gromov** as well as members of the program's organising committee, and two workshops concentrating on different aspects of the program.

The principal topics in which the network will train its young researchers are as follows.

Asymptotic Geometric Analysis John's ellipsoids, Dvoretzky's Theorem, Concentration of measure methods, **Pisier**'s K-convexity theorem, Entropy numbers and Gaussian processes, **Milman**'s isomorphic symmetrisation and its uses. Bourgain's study of the variance of convex bodies using Gaussian processes. Recent work on random and semi-random symmetrisations.

Isometric convex geometry The Brunn-Minkowski theory and Aleksandrov-Fenchel inequalities. Valuations: examples and characterisations. Stochastic geometry: coarea formulas and kinematic formulas. Random and optimal approximation by polytopes: limit shapes for random bodies. Specific algorithms in computational convexity and tomography.

Asymptotic Combinatorics The **Szemerédi** regularity lemma and **Gowers**' use of harmonic analysis in Ramsey theory. Graph eigenvalues, spectral graph theory. Algebraic and geometric expanders and the zigzag product. Applications of probabilistic methods to Ramsey theory and extremal combinatorics. Discrete random structures and their asymptotic behavior. Algorithmic applications of graph eigenvalues and related parameters.

Randomised computation and complexity Impossibility of solving certain problems by deterministic algorithms in polynomial time, Elekes' theorem, **Bárány-Füredi** theorem. Use of oracles in theoretical computer science. Polynomial time solution of the same problems using randomized algorithms, Broder's method and the Jerrum-Sinclair theorem. Algorithm of **Dyer**-Frieze-Kannan, and its improvements for example by Kannan-**Lovász-Simonovits**. Markov Chains, Multiphase Monte Carlo, product estimators. Connection to Cheeger's inequality (conductance) and isoperimetric inequalities.

High-dimensional phenomena in Mathematical Physics Basic results on asymptotic eigenvalue distribution of random matrix ensembles. Techniques using orthogonal polynomials, Markov operators and their relationship to functional inequalities including heat kernel bounds. Recent results on local asymptotic regimes for eigenvalue distributions. Introduction to self-averaging and phase transitions in disordered systems, including recent results of **Talagrand**, together with basic theory of stochastic processes and martingales used therein.

Isoperimetric principles in Probability and Geometry Deviation estimates in pure probability. Heat kernel methods and relationships between logarithmic-Sobolev inequalities, Poincaré inequalities and transportation cost inequalities. Optimal transportation as a tool for symmetrisation. Entropy growth and its relation to geometry. **Talagrand**'s theory of majorising measures (which he has recently presented in a new and accessible form).

All the training and transfer of knowledge mechanisms will be discussed widely within the network to eliminate duplication and guarantee access. Relatively little infrastructure is needed for most mathematical research: computing facilities are certainly adequate at each of the nodes. All teams have older personnel who not only have research experience but have wide experience in supervising research by younger mathematicians, with the possible exception of the Spanish team in which several of the members most central to the project are under 40. For this reason it is proposed that several senior mathematicians from elsewhere in the network should visit Spain for periods of a month or so to ensure that the team is fully integrated.

We are requesting 165 person-months of support for young researchers and 132 for more experienced researchers. These numbers are realistic estimates of the likely demand for places from candidates sufficiently strong to merit support. The balance is slightly in favour of young researchers. In mathematics, the period in which researchers acquire breadth as opposed to depth of knowledge is concentrated in the few years after a doctorate and it is anticipated that competition for the more experienced positions will be fierce. One of the main reasons for this programme is to retain talented young scientists in Europe and halt the drain to the US. There are two moments in a scientist's career when he or she is most likely to move: the start of a PhD or just after it. In the fields covered by this proposal the second is probably the more crucial moment. Many of the leading researchers in these fields are in Europe so that they can attract excellent students: but to keep those students after they have received PhDs, when financial incentives attract them to the US, is exceptionally difficult. We are therefore asking for a considerable proportion of funds to cover postdoctoral researchers.

The individual Career Development Plans for early-stage and more experienced researchers will be compiled by the fellows and their supervisors/mentors. The network will provide the following resources to aid and encourage this process:

- A list of monographs, surveys and recent research papers;
- A list of seminars, workshops and conferences run by the network and by other mathematicians in related fields;
- A web-based file of the network sponsored positions on offer;
- A web-based file of career possibilities;

Mentors will be expected to discuss these opportunities with their students and visitors. One of the most important parts of each fellow's involvement will be attendance at the annual 3-day workshop at which all fellows will be asked to present their recent work. This will considerably enhance their visibility to senior members of the network.

The needs of young and more experienced researchers are different but overlapping. Young researchers need guidance not only in what to learn about the basics of their own field but also help in selecting the area in which they want to work after their PhD. More experienced researchers need to be kept acquainted with recent developments in their own and related fields and to have the opportunity to collaborate actively in research with other mathematicians. Both groups can be well served by meetings and visits to other institutions but it should be borne in mind that meetings should provide expository lectures for the younger researchers and announcements of new results aimed more at the older researchers, while the nature of a visit is different for the two groups. A student who visits a host institution should be given research supervision to at least the same extent as in his/her own university and must be personally introduced to faculty members by the person s/he is principally visiting. For more experienced researchers, the most important feature of a visit will be actual research collaboration with someone in the host institution.

The most important complementary skill for any scientist and especially mathematician is that of communication. Senior staff will encourage their students to present their work and that of others at seminars in their own universities.

Excluding graduate students, the network has a 12% participation by women. This is fairly high by the standards of mathematics. The proportion of women is particularly high in Italy and Greece and this may reflect local conditions, but the appearance of genuinely successful women in several teams almost certainly testifies to the friendly atmosphere for which these areas of mathematics are well-known: it seems likely that one of the main reasons for the under-representation of women in science is that women are more suspicious of work environments based on aggressive competition rather than cooperation.

As explained in section 4.1 there will be a senior female team member who will coordinate contacts between the network and Associations of Women Mathematicians so as to provide the support structures that can be lacking in a predominately male environment. The fact that *all* younger researchers will be expected to present their work at annual workshops will ensure that no bias can enter the selection of these speakers.

One of the teams from Eastern Europe has drawn attention to the fact that many women are concerned that an explicit concentration on gender issues may lead to the suspicion that women have succeeded only as a result of special treatment. The network coordinators are sensitive to both sides of this issue.

B2.2. Impact of the training and/or transfer of knowledge programme

The subjects involved in this project lie at the crossroads of crucial areas of mathematics and physics: probability theory, geometry, statistical physics, combinatorics and computation. In many areas of science, the emphasis is moving toward the analysis of complex systems which exhibit random-like behaviour owing not to explicitly random components but simply to their complexity. The mathematical theory needed to analyse and even just to simulate such systems will need to invoke insights from geometry and combinatorics as well as physics and probability theory. Many of the most dramatic mathematical achievements in these directions have come from the teams in this program.

In several areas of the programme (convex geometry, deviation inequalities, combinatorial phase transitions) Europe is already significantly stronger than its main competitor, the US. While the drain of established scientists to more lucrative positions in the US is well understood and easily visible, a perhaps far more serious problem is the disappearance of talented younger researchers who never return. Europe can be made more attractive to doctoral students and postdoctoral researchers by identifying areas of research strength and providing students and post-docs in those fields with the wherewithal to visit leading researchers other than their immediate supervisors.

The principal benefits accruing to participants will be access to a broader vision of the mathematical framework within which they belong and to the expertise of a much larger number of senior scientists, able to provide a wide variety of different contributions to their training. Among other things, such contacts can help to avoid duplication of work by researchers in differing fields.

The main need for the transfer of knowledge between different teams and even between individuals within the same team is that the participants come from branches of mathematics that have historically been considered quite separate. The fact that high-dimensional phenomena have become so important in all these areas over the past few years has several causes, but the underlying reason is now fairly clear. Physical systems and computer networks exhibit random-like behaviour and need to be analysed on that basis, but they do not satisfy the very precise independence assumptions that underly the classical principles of probability theory. This means that the classical techniques (analyse the 1-dimensional case and then multiply to get the high-dimensional case) will not work. These systems have to be treated as high-dimensional from the start. This in turn means that it is essential to have a high-dimensional and therefore geometric "picture".

Europe as a whole has the capacity and expertise to develop a generation of young researchers who are at home with all the fields in which this new viewpoint is relevant, in a way that no single European country could hope to do. The enhanced potential of this younger generation can be expected to have a profound influence on the development of this aspect of science, internationally as well as nationally and locally.

The two principal methods by which the network will enable the transfer of knowledge especially to younger members will be through the visits by early-stage and more experienced researchers to institutions other than their own and the annual meetings and workshops which bring together personnel in all areas of the program. In addition, the network will maintain a reference list of recent work by its members and others, available in electronic form from the website, and circulate a list of monographs and expository articles on the topics of the program. Senior personnel will be encouraged to write expository articles in a style accessible to as many as possible, in connection with the activities of the network.

The training and transfer of knowledge provided by the network will significantly assist researchers in the development of their future careers. One of the most serious problems that can damage the career of a young mathematician is that s/he becomes trapped in a scientific backwater. In order to select a field for research after one's doctorate it is necessary to have exposure to a variety of different influences and to identify the fields in which progress is being made in significant directions. If a young mathematician does not see this variety during or soon after his/her doctoral studies, s/he may find that the pressure to publish makes it too risky to switch fields (with the attendant need to spend time learning new techniques). This problem is especially acute for women since its timing may well coincide with the arrival of children.

A more mundane, but important, respect in which this kind of programme can help young mathematicians is that it will introduce them to a significant number of senior figures in several fields. This greatly increases their chances of obtaining jobs at leading institutions simply because there is a better chance that their names will be known to senior people at those institutions.

Finally, and ultimately most importantly, this programme will enable its younger members and, one hopes, its older ones to produce more significant, timely and well-presented research than without it. The maturity that comes with a broad mathematical perspective and the enthusiasm generated by being surrounded by exciting research in several fields are what turn talented young people into the leaders of the future.

The network members are currently supervising over 100 research students.

B2.3. Planned recruitement of early-stage and experienced researchers

The network plans to provide 165 person-months of support for early-stage researchers and 132 person-months of support for more experienced researchers. The balance is therefore 5:4.

Appointments for early-stage mathematicians will be targeted mainly at PhD students within the network: given the time pressure under which most PhD students operate, it is unlikely that many students working in more distant disciplines not covered by the network, would feel that they had time to take up such training positions. The timing and length of training provided to young researchers will be carefully adapted to the needs of each student's thesis preparation. Appointments for more experienced researchers will, we hope, be of interest to a wider group of mathematicians but members of network teams will be strongly encouraged to apply.

Vacancies will be advertised via the network mailing list, on the departmental home pages of the nodes and the network homepage. The EU website itself has proved to be effective in recruiting early-stage researchers for other programs administered by individuals from the network. The adverts will describe the research area, type(s) of fellowships and conditions of eligibility.

The main criteria for choice of early-stage researchers will be research potential as witnessed by supervisors' references or work submitted by the applicant and compatibility with the programme of the network. The main criteria for more experienced researchers will be the quality of published work, references from leading scientists and compatibility. The applications will be evaluated by at least two members of the appointing team in consultation with the network coordinator or team coordinator (whichever is more appropriate). All appointments will be scrutinised by the appointments subcommittee of the steering committee to ensure co-ordination between teams and to monitor the gender balance of applicants and appointees. It is assumed that the typical length of visits of early-stage researchers will be up to 3 months and of more experienced researchers, 6 months. The network has requested (on the A4 form) a significant number of person months of support at those nodes where summer or winter schools will be held.

The network will pursue an active equal opportunities policy: appointments will be made without regard to race or gender. The network will ensure that women from the teams are actively engaged in the evaluation process and this will be made clear in the advertisements. Advertisements will be sent to Associations of Women in Mathematics in order to encourage as balanced an application pool as possible.

We do not expect any difficulties in recruiting young researchers for this program, although the political situation might deter some visits to Israel. However, the excellence of the senior personnel from its different nodes in Tel Aviv, Haifa and the Weizmann Institute, their wide expertise in most of the fields of this programme and their exceptional reputation as supervisors/mentors makes it a very attractive destination and should outweigh the political situation.

Because of the growing importance of the study of high-dimensional phenomena in science, and because of the considerable number of highly talented younger people already attracted to this field, it is vital that we offer them the best possible training that we can provide: this programme will provide them with access to a very broad range of expertise at the highest level.

The ratio between the number of person months of appointment for early stage researchers and the number for more experienced researchers is $165 : 132 = 5 : 4$. It is essential that there should be plenty of opportunities for postdoctoral researchers because there are already in the network more than 100 PhD students, many of whom will soon be candidates for a post-doc position. As mentioned earlier, the attraction of the US for post-doctoral positions in mathematics is enormous. Europe will not be able to keep the best young researchers, if we do not offer them the possibility of developing their research beyond their PhD.

As already explained in section 2.1, several extremely successful programmes (trimester or semester) with strong educational content were organised by members of the network during the past few years. These will continue: the next one is scheduled in 2005 at Schrödinger Institute in Vienna and there will be a trimester programme at IHP in Paris in 2006. As were the previous ones, in Berkeley (MSRI), Vancouver (PIMS) and Edinburgh (ICMS), it is clear that such programmes will be well attended by students and postdoctoral researchers from the proposed network. Each of these programmes will bring together researchers in all the main areas of the project, and especially, many of the senior personnel. For example, many of the senior personnel are already scheduled to pay long term visits to the IHP programme in spring 2006. The presence of so many senior personnel in one place represents a perfect opportunity for the training of early-stage researchers: so we are asking for many more person-months of support at these particular times and places.

	Early-stage and experienced researchers to be financed by the contract			Other professional research effort on the network project	
Network Team	Early-stage researchers to be financed by the contract (person-months) (a)	Experienced researchers to be financed by the contract (person-months) (b)	Total (a+b) (c)	Researchers likely to contribute (number of individuals) (d)	Researchers likely to contribute (person-months) (e)
1	24	24	48	16	255
2	24	18	42	19	182
3	9	6	15	7	230
4	18	12	30	12	110
5	12	6	18	11	168
6	9	6	15	7	110
7	9	6	15	13	163
8	12	6	18	10	139
9	6	6	12	6	115
10	9	12	21	11	139
11	9	6	15	7	120
12	24	24	48	14	216
13	0	0	0	5	82
Totals	165	132	297	138	2029

B3. QUALITY/CAPACITY OF THE NETWORK PARTNERSHIP

B3.1. Collective expertise of the network teams

The network contains 13 teams. It includes around 140 researchers and has more than 100 research students currently. Each team is a legal entity consisting of researchers from various universities grouped around a strong local group. France and Germany have two teams, each of which has a strong group of researchers as well as the infrastructure and experience needed for training. This division will improve the management structure of the network. Team 13 from Ukraine, is from a Third Country. Its participation is extremely important because of its great expertise in the mathematical physics part of the programme: this is explained in detail in section 1.

Members of all teams have very considerable experience in mentoring research students. Indeed, about a quarter of the personnel in each team are former students of older members of the team and many of these former students have became distinguished in their own right.

The percentage of individual involvement has been estimated as the percentage of full time employment. Taking into account teaching, administration and research in a typical university, a percentage of **30%** is considered as **full involvement** for a participant in this programme. Exceptions were made for local coordinators and scientists in charge who may be involved from 40 to 50%.

The quality of the network's personnel can be judged from the fact that between them they have won 2 Wolf prizes, a Fields medal, 2 Ostrowski prizes, a Salem prize, a Fermat prize, a Pólya prize, 2 European prizes and numerous national prizes and awards. In total, they hold more than 140 memberships of editorial boards of international mathematics journals and 11 are the managing editors of such journals.

To begin with we shall describe the complementarity and summarize the collective expertise of the teams in the network. Over the last couple of decades there has been a steady stream of collaborations between individuals from different parts of the network: some of these are detailed in section 3.3. More importantly, several major mathematical developments, spread over many years, have involved significant contributions at different times from different teams. In several cases the complementary expertise of different teams was essential for the furtherance of these developments.

From the mid-seventies onwards a combination of probabilistic and analytic methods led to a progressive understanding of the local structure of normed spaces. The main contributors were **Figiel, Giannopoulos, Gromov, König, Kwapień, Maurey, Milman, Pajor, Pisier, Schechtman and Szarek** from teams 1, 3, 5, 6, 12 of the network and Bourgain, Johnson, Lindenstrauss, and Tomczak-Jaegermann from outside it. The emphasis lay on the influence of increasing dimension on geometric properties. A central role was played by the high-dimensional concentration phenomenon, familiar for many years in probability but pioneered in geometry by **Gromov and Milman**, which revealed a completely new significance to classical geometric principles such as the isoperimetric inequality.

The geometric view of probabilistic concentration inspired the remarkable results of **Talagrand** (team 1) in the mid-90's, demonstrating new deviation principles for very general probabilistic structures. Following this and the work of **Bakry** and Emery, **Ledoux** (team 10) found a way to link concentration estimates with entropy inequalities coming from statistical physics and information theory (logarithmic Sobolev inequalities) and from the theory of Markov chains (transportation cost inequalities).

Optimal transportation of measures was pioneered by **Brenier** in the context of PDE's but was found by **Barthe, Cordero** and others to be the perfect tool for understanding classical results in harmonic analysis which had in turn been used by **Ball** to solve purely geometric problems. More recently, transportation was the starting point for the solution, by **Ball, Barthe** and two students from the Israeli team, of a long-standing problem on entropy growth in information theory.

Isoperimetric principles were also essential to the development of randomised polynomial time algorithms by **Brieden, Dyer, Gritzmann, Lovász, McDiarmid, Simonovits, and Welsh** from teams 2, 4, 11 of the network and Applegate, Frieze, Jerrum, Kannan and Sinclair from outside. It is vital for the effectiveness of these algorithms that random walks (and more general Markov processes) rapidly diffuse into equilibrium configurations: that they are "rapidly mixing". **Welsh** has used randomised algorithms to study the partition functions for several standard models in statistical physics.

During the last ten years the theory of random matrices has become very exciting. Members of the network were heavily involved in this development from the late 60's, when Marchenko and **Pastur** found the eigenvalue distribution of the generalised Wishart ensemble, till now, when a number of pioneering and often quite complete results were obtained by **Alon, Blower, Füredi, Krivelevich, Komlós, Latała, and Szarek** from teams 1, 2, 4, 6 and 12 on extreme eigenvalues, and by **Pastur** and **Sherbina** from team 13 on universality. Statistical physics is a branch of theoretical and mathematical physics where high-dimensional analysis is indispensable. Of particular interest are spin glasses and neural networks both in their own rights and because of their numerous links with probability, optimisation, combinatorics, biophysics, and so on. Some of the first rigorous results were those on the self-averaging in basic models obtained by **Pastur** and **Shcherbina** in the early 90's. Within the modern development there has been a recent breakthrough by **Talagrand** on the free energy in the Sherrington-Kirkpatrick model.

The rise of combinatorics as a mathematical discipline over the last 50 years, has impinged upon and fed into many parts of functional analysis, probability theory, geometry and computing. The Budapest school founded by Erdös, and comprising **Bárány, Füredi, Lovász, Sós, Simonovits, Szemerédi**, Komlós and others, the Israeli group (**Alon, Krivelevich**) and the Cambridge group (**Gowers, Green, Leader, Riordan**) established by **Bollobás**, are among the undisputed world leaders in combinatorics.

One of the most famous results in combinatorics is the theorem of **Szemerédi** which guarantees the existence of long arithmetic progressions within dense sequences of integers. The original proof was combinatorial and introduced the "regularity lemma" which states in a precise way, that all complex networks can be modelled to a large extent by random ones. The "regularity lemma" was turned into an effective computational algorithm by **Alon** and Duke, Lefmann, Rödl, Yuster. In 1998 **Gowers** found a way to use harmonic analysis to improve enormously the quantitative bounds in **Szemerédi's** theorem.

Finally, classical convex geometry and random methods have come together in the study of approximation of general convex bodies by polytopes, by **Bárány, Buchta, Gruber, Larman, Reitzner, Schneider, Schütt and Werner** from teams 1, 2, 3, 4, 9, 11. It turns out that random approximation is almost exactly best possible. As part of this study **Bárány** and Vershik discovered the remarkable fact that in many natural situations random polytopes have a typical or limiting shape: an astonishing degree of uniformity, given the huge variety of possible shapes. A central limit principle for these limit shapes was subsequently found by Sinai and others using techniques from ergodic theory.

TEAM 1 Centre National de la Recherche Scientifique (CNRS) coordinator: A. Pajor

---

Subnodes: Universities of Pierre et Marie Curie (UPMC), of Créteil (UC), Lille-1 (UL1), Ecole Polytechnique (EC), Institut IHES.

KEY STAFF: Professors: F. Barthe (30%, CNRS), P. Cattiaux (30%, EC), M. Gromov (30%, IHES), C. Houdré (30%, UC), B. Maurey (30%, CNRS), M. Meyer (50%, CNRS), A. Pajor (50%, CNRS), G.Pisier (30%, UPMC), S. Szarek (30%, UPMC), M. Talagrand (30%,UPMC,CNRS), E. Werner (30%, UL1). Ass. Professors: D. Cordero-Erausquin (30%, CNRS), M. Fradelizi (30%, CNRS), O. Guédon (40%, UPMC), Cyril Roberto (30%, CNRS), P.-M. Samson (30%, CNRS).

Research interest

The team covers a research group from around Paris, mainly located at Marne-la-Vallée and at the Institut Mathématiques in Paris. It employs 16 researchers. Its main expertise covers a wide range of subjects in functional analysis, convexity and in probability. It is one of the leading centers in probability, concentration of measure and empirical processes. It has a long history of collaboration, joint workshops, joint papers or bilateral research programmes, with the teams from London, Tel Aviv, Kiel, Warsaw, Greece, as well as strong relations and cooperation with the teams from Toulouse, Spain and Hungary.

Principal research personnel

F. Barthe has pioneered the use of optimal transport to derive functional geometric inequalities. He was recently awarded the Peccot lecture at Collège de France. Jointly with Artstein (team 12), Ball (team 3) and Naor they solved an old conjecture by Shannon about entropy increase in the Central Limit Theorem. He will move next year to Toulouse (team 10).

P. Cattiaux is a well known expert in probability. He recently worked on refinements of Talagrand's results on exponential measures.

M. Gromov is one of the greatest mathematicians of our time, who contributed, among many other things, to the theory of High Dimensional Phenomena through his study of the concentration property in joint works with Milman (team 12) and also in the 100 page chapter of his recent Monograph "Metric Structures for Riemannian and Non-Riemannian Spaces". He aims to contribute to the success of the Programme through the lectures he will deliver in our Educational Workshops.

C. Houdré is a well known expert in probability, deviation and concentration inequalities.

B. Maurey is a world-renowned expert in all aspects of functional analysis. Together with W. T. Gowers (team 2) he solved the famous unconditional basic sequence problem posed by Banach in 1933. He pioneered the use of martingales in the study of concentration. His recent work concerns transportation of measures and its connection with convexity.

M. Meyer is a well known expert in convex geometry, both classical and asymptotic, and in approximation of convex bodies by polytopes.

A. Pajor is one of the leading experts in the local theory of normed spaces. He initiated the study of isotropicity and his joint work with Milman (team 12) is a standard reference which was the origin of much of the work in both in Convexity and in Asymptotic Theory. He transferred (in the joint work cited below) results developed originally for normed spaces to the Asymptotic Convexity setting.

G. Pisier is a full member of the French Academy of Sciences and winner of numerous prestigious prizes. He is a mathematician of extremely broad interests, a dedicated educator, a prolific writer and has supervised many doctoral dissertations and organised numerous conferences. His interest in this programme is connected with his lasting contribution to Asymptotic Geometric Analysis and his more recent work related to random matrices.

S. Szarek has pioneered the use of volumetric methods in the geometry of Banach spaces. He is a leading expert in various areas of high-dimensional convexity, particularly in probabilistic and combinatorial aspects of the theory.

M. Talagrand is recognized as one of the world's leaders in theoretical probability. He was a plenary speaker at the ICM in 1998 and has solved several very famous problems including the recent solution (May 2003) to the Parisi conjecture (on the free energy of the Sherrington-Kirkpatrick model). His seminal work on Gaussian processes revolutionized the field, and his deep and prolific results on the concentration phenomenon opened up new perspectives in diverse areas of pure and applied mathematics. This body of work represents over 500 pages of publications in refereed journals, and a recent 600 page book.

E. Werner is an expert in convex geometry. Her recent work with team 3, showing that random approximation of convex bodies by polytopes is essentially optimal, shocked the experts.

A group of brilliant young permanent researchers includes **D. Cordero, M. Fradelizi, O. Guédon, C. Roberto**.

Mentoring, transfer of knowledge, facilities and infrastructure

The team has a large experience in training and in the mentoring of young researchers, and has an excellent record of successful PhD students. It has currently 6 thesis students, 4 early-stage researchers and 2 post-docs. It organizes a regular weekly seminar and a "groupe de travail" for young researchers. The team organizes more than 2 workshops each year at Marne and Paris (3 workshops and 1 international conference this academic year). Located at Marne-la-Vallée or at the Institut Mathématiques in Paris, the infrastructure, library and facilities available to young researchers are excellent. The team holds 14 memberships of editorial boards of international mathematics journals and 1 managing editor of such journals.

Two significant publications:

1. M. Talagrand: Self-organisation of a spin glass model at low temperature. Reviews in mathematical physics 15, 2003, 1-78.
2. V. Milman and A. Pajor: Regularization of star bodies by random hyperplane cut off, to appear in Studia Math. 2003.

TEAM 2	**University College London (UCL)**	**coordinator: K. Ball**

Subnodes: University of Cambridge (UC), University of Edinburgh (UE), University of Lancaster (ULA), University of Leeds (ULE), University of Oxford (UO).

KEY STAFF: K. M. Ball (50%, UCL), I. Bárány (20%, UCL), M. Csörnyei (20%, UCL), D. G. Larman (20%, UCL), L. Parnovski (10%, UCL), D. Preiss (30%, UCL), A. Scott (30%, UCL), B. Bollobás (15%, UC), W. T. Gowers (20%, UC), B. Green (30%, UC), I. B. Leader (25%, UC), O. Riordan (25%, UC), A. Carbery (5%, UE), J. Wright (5%, UE), G. Blower (30%, ULA), M. Dyer (10%, ULE), C. McDiarmid (15%, UO), D. Welsh (10%, UO), M. Anttila (10%, UCL)

Research interest

The British team brings a wide variety of expertise to the project, covering most areas of the program. The central node at University College London employs 7 researchers, mainly in the geometric and analytic aspects of high-dimensional phenomena. There are many links between these researchers and those from the Paris and Toulouse nodes. A large British subnode, located in Cambridge, is one of the world's leading combinatorics centres. This group has very close connections with almost all members of the Hungarian team. The work of Dyer, McDiarmid and Welsh is also closely connected with the work of Lovász and Simonovits from the Hungarian team, on randomised algorithms. Bárány and Larman's work is complementary to that of Schütt from the German team, Werner from the French and Gruber and Reitzner from the Austrian team.

Principal research personnel

K. Ball has pioneered the use of probability theory and harmonic analysis to solve problems in high-dimensional geometry. He and F. Barthe from the French team, together with two students, recently found a solution of the notorious problem of showing that the entropy of normalised sums of independent, identically distributed random variables, increases with the number of summands. He has just been awarded a Royal Society Leverhulme, Senior Research Fellowship.

I. Bárány is an acknowledged expert in combinatorial and convex geometry. In recent years, he has worked extensively on problems related to the limiting shape of randomly chosen bodies and to the optimal rate of random approximation of geometric objects and on topological methods in combinatorics, about which he recently gave a lecture at the ICM.

G. Blower began his career in the field of functional analysis but has moved into the theory of random matrices: he recently gave a striking new demonstration of the convergence in distribution of eigenvalues of random matrices (one of the crucial planks of the theory) under hypotheses much more general than the existing ones: the article has recently appeared in the journal of statistical physics.

B. Bollobás is one of the most famous combinatoricists in the world. He has not only been immensely influential in the development of random combinatorics through his own research, but has supervised nearly 40 research students, most of whom have gone on to pursue very successful careers in mathematics. 5 of them are senior researchers in this project.

T. Carbery is Britain's leading harmonic analyst. With Jim Wright he recently determined the optimal L_p estimates for polynomials on convex bodies and he has been one of the main advocates of the new links between harmonic analysis and combinatorial number theory in the directions pursued by Bourgain, Gowers and others. He was the driving force behind the immensely successful training workshop for students and postdocs held under the auspices of the ICMS in 2002.

M. Csörnyei, former student of David Preiss, with similar interests to his, is one of the world's leading young women mathematicians. She recently won a prestigious Royal Society Wolfson Research Award: only the second mathematician to do so.

M. Dyer is an expert on computational algorithms; particularly randomised ones. He was one of the original group who constructed a randomised polynomial-time algorithm to calculate the volumes of convex bodies.

T. Gowers won a Fields medal for his work on the geometry of Banach spaces and since then has worked in combinatorial number theory using methods from harmonic analysis. Some years ago he produced startling new estimates for the density of a sequence needed to guarantee that it contains long arithmetic progressions: an enormous quantitative improvement upon the famous theorem of Szemerédi. He was a prize fellow of the Clay Institute in 2001 and 2002.

D. Larman is a leading figure in all aspects of convex geometry. Recently he has worked with Bárány on random approximations of convex bodies by polytopes. Currently head of department at UCL he has a wide range of administrative experience and has mentored many students.

I. Leader is an expert on combinatorial isoperimetric problems and Ramsey theory. He has collaborated with Barthe from the French team as well as many members of the British and Hungarian teams. He recently won a Whitehead prize of the London Mathematical Society.

C. McDiarmid is well known for his work in asymptotic combinatorics and computational algorithms.

L. Parnovski is well known for his work in the spectral theory of linear differential operators and in particular for his contributions on the famous Bethe-Sommerfeld conjecture.

D. Preiss is the world expert on Lipschitz maps and their connections to geometric measure theory. He is renowned for his solutions of differentiability (linearisation) problems for Lipschitz maps, which typically require a deep study of geometric descriptions of negligible sets. Many basic difficulties of the theory can be traced back to the (limit case of the) concentration phenomenon.

A. Scott is well known in the field of combinatorics, in particular in extremal problems on graphs. Recently he and the physicist A. Sokal have begun a detailed combinatorial study of phase transitions in the lattice gas model. Among other things this work aims to provide a unified and more precise analysis of the famous phase transition theorem of Heilmann and Lieb for the monomer/dimer model in statistical physics.

D. Welsh is a leading figure in the theory of randomised algorithms and their uses in complexity theory and computing. His recent work has centred on the computation of values of the partition function for the Potts model in statistical physics.

J.Wright is an expert in classical harmonic analysis and its relations to convexity and combinatorics.

The Cambridge combinatorics group also includes the exceptionally talented young researchers, **O. Riordan** and **B. Green**.

M. Anttila is a former student of Keith Ball, currently employed in Turku, in Finland. She is studying the geometric structure of proteins but retains an interest in high-dimensional geometry in general.

Mentoring, knowledge transfer, facilities and infrastructure

The British team members between them, have supervised over 100 successful PhD students and are currently supervising 23 students. The team members hold 40 editorships of international journals between them, 3 of which are managing editorships.
The UCL node has for the last few years been the recipient of a Marie Curie training site award administered by D. Preiss and involving K. Ball and D. Larman. A dozen students from EU and affiliated countries have each spent 3 months at UCL. These students received high-level training in mathematical analysis.

Two significant publications:

1. K. M. Ball, F. Barthe and A. Naor, Entropy jumps in the presence of a spectral gap, Duke Math. J. **119** (2003), 41-64.
2. W. T. Gowers, A new proof of Szemerédi's Theorem, Geom. Funct. Anal. **11** (2001), 465-588.

TEAM 3 **University of Kiel (UK)** **coordinator: H. König**

Subnodes: University of Jena (UJ), University of Oldenburg(UO).

KEY STAFF: Professors: B. Carl (30 %, UJ), A. Defant (30%, UO), H. König (30%, UK), C. Schütt (30%, UK). Ass. Professors: A. Hinrichs (30 %, UJ), C. Michels (30%, UO), Ch. Richter (30%, UJ)

Research interest

The team covers research groups at Kiel, Jena and Oldenburg. It employs 7 researchers, with currently 4 PhD students. The main expertise is in the "asymptotic analysis of finite-dimensional normed spaces" with its relations to operator ideals, classical convexity theory, spherical designs and approximation theory. Many projects in functional analysis and convexity theory being studied by the groups involve input from probability theory and measure theory. The team has long-standing cooperations with groups in Marne-la-Vallée, Tel Aviv and Warsaw and long contacts with the teams in London and Vienna. Cooperation with the group in Greece is very natural given the topics of the programme.

Principal research personnel

B. Carl is a leading expert in the theory of operator ideals, s-numbers, eigenvalue distribution theory and approximation theory. He has contributed essentially to the estimation of entropy numbers and Gelfand numbers.

A. Defant is a well-known expert in the theory of absolutely summing operators and its extensions and applications. He has made important contributions to the study of Gelfand and Weyl numbers.

H. König is a well-known expert in the local theory of normed spaces, operator ideals, eigenvalue distributions. He contributed in an essential way to the study of projection constants using spherical design techniques for estimates and examples.

C. Schütt is a leading expert in classical convexity theory and Banach space theory. He proved important results on approximating convex bodies by polytopes.

C. Michels is well-known for his work on absolutely summing operators as well as type and cotype.

A. Hinrichs and **Ch. Richter** used spherical designs in their negative solutions of the Borsuk and Knaster conjectures. A. Hinrichs is a well-known expert in Banach space theory using discrete methods, probability theory and topology.

Mentoring, transfer of knowledge, facilities and infrastructure

The team has a long experience in training and mentoring of students and young researchers with successful PhD students. It organizes regular weekly seminars with lectures by local personnel and by foreign visitors. Members of the team have organised various conferences which stimulated research by young mathematicians. Library facilities and further infrastructure are adequate and available to all young researchers.

Two significant publications:

1. A. Hinrichs, Ch. Richter; New sets with large Borsuk numbers, to appear in Discrete Math.
2. C. Schütt, E. Werner; Polytopes with vertices chosen randomly from the boundary of a convex body, Lecture Notes in Math. 1807, Springer 2003, 241-422 (ed. V. Milman, G. Schechtman).

TEAM 4 Rényi Institute of Mathematics (RI) coordinator: M. Simonovits

---

Subnodes: Eötvös Loránd University, (ELTE)

KEY STAFF: Professors: I. Bárány (30%, RI/UCL), Z. Füredi (20%, RI), M. Laczkovich (30%, ELTE/UCL), L. Lovász (30%, ELTE/Microsoft), E. Makai, Jr (30%, RI), K. Marton (30%, RI), János Pach (30%, RI), M. Simonovits (50%,RI), V. T. Sós (30%, RI), E. Szemerédi (30%, RI). Ass. Professors: K. Böröczky (Jr) (30%, RI), B. Csikós (30%, ELTE).

Research interest

The strength of the Budapest group is in combinatorial methods and their applications in other fields. Using this, the Hungarian team brings special expertise to the project covering the following areas: extremal combinatorics, random structures and quasi-random structures, comparison of deterministic and random structures, design and analysis of algorithms (in computer science and geometry), discrete and computational geometry. An important feature of the Budapest school is the frequent and successful use of random and geometric methods in combinatorics: the Lovász local lemma, Szemerédi's regularity theorem, the Erdős-Simonovits hypergraph theorem, the Kneser-Lovász theorem. The Hungarian team has very strong connections with the French team (asymptotic properties of convex bodies), with the British team (hypergraphs, probabilistic approach), and with the German and Austrian teams (in convexity theory).

Principal research personnel

I. Bárány is one of the leading mathematicians in discrete geometry and convexity. He pioneered the use of algebraic topology in geometry. He was an invited speaker at the ICM in Beijing, 2002.

K. Böröczky, Jr is a talented young researcher, nicely combining several classical and modern areas of geometry.

B. Csikós is a very talented mathematician, of extremely wide knowledge. His results on the classical Hadwiger-Kneser-Poulsen conjecture have lead to its full solution in the planar case.

Z. Füredi was an invited speaker at the ICM in Zürich, 1994. He is a leading expert in several areas of combinatorics, among others in those connected to set-intersection theorems, to the connection of block designs and graph theory, extremal graph theory.

L. Lovász, winner of the Wolf prize, is one of the world's leading experts in combinatorics. His research topics include combinatorial optimization, algorithms, complexity theory, random walks on graphs, semidefinite programming combinatorics, and also the theory of algorithms. Among his many important contributions to the field are the proof of the (weak) perfect graph conjecture, the proof of Kneser's conjecture on the chromatic number of Kneser graphs (using deep topological methods in combinatorics), the proof of Shannon's conjecture on the Shannon capacity of C_5. Several methods in combinatorics established by him have become crucial and by now everyday tools: the Lovász Local Lemma, the Lenstra-Lenstra-Lovász algorithm and extensions of the ellipsoid method. He was a plenary speaker at the ICM in Kyoto, 1990, and an invited speaker at the ICM in Warsaw, 1983.

M. Laczkovich's main interest lies with analysis and measure theory. He became world famous by solving the old conjecture of Tarski on how to square the circle for which he received the Ostrowski Prize in 1993.

E. Makai is a bright and talented mathematician, of very wide knowledge, well versed in topology, functional analysis, discrete mathematics, and discrete geometry.

K. Marton is working in information theory and in discrete mathematics. Her contributions are related to the Shannon capacity and its relation to perfect graphs.

J. Pach is one of the leading figures in computational geometry, and an acknowledged expert in the field of geometric graphs, their crossing numbers, and the ways they can be drawn in the plane.

M. Simonovits works in discrete mathematics. A joint result of Erdős and Simonovits is the supersaturated hypergraph theorem that has been applied hundreds of times. He also worked together with Lovász (and R. Kannan) on randomized algorithms to estimate the volume of high dimensional convex bodies.

V. T. Sós is one of the leading figures of the famous Hungarian combinatorial school. Her main research fields are: combinatorics and graph theory, number theory, uniform distribution, random and pseudo-random structures, Ramsey Theory and Ramsey-Turán theory.

E. Szemerédi's works have had great impact on several fields in mathematics: graphs and hypergraphs, number theory, and theoretical computer science. He proved the famous $r_k(n) = o(n)$ conjecture (Erdős-Turán) stating that sequences of integers with positive lower density contain arbitrarily long arithmetic progressions. The so called Szemerédi regularity lemma has revolutionized the asymptotic theory of combinatorics. Szemerédi, together with Ajtai and Komlós, solved a famous conjecture of D. E. Knuth on parallel sorting. He was an invited speaker at the ICM in Vancouver, 1974.

Mentoring and transfer of knowledge, facilities and infrastructure

The team has a large experience in training and in the mentoring of young researchers and has an excellent record of successful PhD students. Füredi, Lovász, Pach, Szemerédi have several PhD students at all times. We regularly organise international conferences, workshops and mini-congresses. The infrastructure, library, lecture rooms, computers, and all other facilities at the Alfréd Rényi Institute of Mathematics and at Loránd Eötvös University in Budapest are available for all researchers. The team members hold more than 40 editorships of international journals between them, 2 of which are managing editorships.

Two significant publications:

1. R. Kannan, L. Lovász and M. Simonovits: Random walks and an $O^*(n^5)$ volume algorithm for convex bodies, Random Structures Algorithms, **11** (1997), 1–56.
2. I. Bárány, A. Por: On $0-1$ polytopes with many facets, Advances in Math., **161** (2001), 209–228

TEAM 5 **University of Crete (UOC)** **coordinator: A. Giannopoulos**

Subnodes: Universities of Athens (UOA), of the Aegean (UAEG), Agr. Univ. of Athens (AUA).

KEY STAFF: C. Athanasiadis (30%,UOC), L. Dalla (40%,UOA), V. Felouzis (30%, UAEG), D. Gatzouras (30%, AUA), A. Giannopoulos (50%, UOC), M. Hartzoulaki (30%, UOC), M. Kolountzakis (30%, UOC), I. Kyrezi (30%, UOC), M. Papadimitrakis (30%, UOC), I. Perissinaki (10%, UOC), A. Tsolomitis (40%, UAEG).

Research interest

The team covers research groups from Heraklion, Athens and Samos. The main expertise of the team is in asymptotic geometric analysis, classical convexity, applications of Fourier analysis to convex geometry, combinatorics and probabilistic methods. Several members of the group have strong links and research collaboration with the teams from Tel-Aviv, Paris and London. In the last years, leading experts (including Milman, Olevskii, Pelczynski and Schneider) made long-term visits to Heraklion and gave series of lectures as "distinguished Pichorides lecturers".

Principal research personnel

C. Athanasiadis works in combinatorics and its interaction with algebra, discrete geometry and topology. More specific research interests include enumerative combinatorics, matroid theory, reflection groups and Weyl groups, combinatorics of convex polytopes, arrangements of hyperplanes and topology of simplicial and cell complexes.

L. Dalla is an expert in classical convexity. She has made significant contributions to the boundary structure of convex bodies, approximation of convex bodies by random polytopes, discrete geometry and volume inequalities. Her research interests also include fractal geometry and Banach algebras.

D. Gatzouras works in probability and ergodic theory. Recently, he studied the asymptotic behaviour of n-fold convolution of a probability measure and obtained an analogue of the Beurling-Gelfand spectral radius formula for the Fourier transform of L^1-functions on compact groups.

A. Giannopoulos has contributed to classical and asymptotic convex geometric analysis. His research interests include the geometry of the Banach-Mazur compactum, isomorphic theory of high dimensional convex bodies, isotropic positions of convex bodies and related extremal problems, volume estimates and inequalities involving mixed volumes.

M. Kolountzakis' mathematical interests range from harmonic analysis and its applications to geometric and number-theoretic problems, to probability theory and theoretical computer science. His current research deals with spectral and tiling properties of convex bodies and extremal problems concerning the size of positive definite functions whose support is restricted to be in a convex body.

M. Papadimitrakis is an expert in complex analysis, harmonic analysis and its applications to convex geometry. He has contributed to the solution of the Busemann-Petty problem, the Steinhaus tiling problem and the spectral properties of high dimensional polytopes (with M. Kolountzakis).

A. Tsolomitis is an expert in convex geometry. He has made significant contributions to convolution bodies, isomorphic symmetrization, random polytopes and volume estimates.

The group also includes the younger researchers **V. Felouzis**, **M. Hartzoulaki** (currently a post-doctoral fellow at the University of Missouri, Columbia), **I. Kyrezi** and **I. Perissinaki**.

Mentoring, knowledge transfer, facilities and infrastructure

The team has experience in supervision of doctoral theses. Currently, there are four early stage researchers and five PhD students. The Heraklion group has a very active weekly seminar in analysis and has organised several working seminars, especially in the areas of asymptotic convex geometry, Fourier analysis techniques in convex geometry and combinatorics. The infrastructure, library and facilities available to young researchers in Crete, Athens and Samos are quite good.

Two significant publications:

1. A. Giannopoulos and V. D. Milman, *Extremal problems and isotropic positions of convex bodies*, Israel J. Math. **117** (2000), 29-60.
2. M. Kolountzakis, *Distance sets corresponding to convex bodies*, to appear in Geom. Funct. Analysis.

TEAM 6 Instytut Matematyczny PAN (IM PAN) coordinator: P. Mankiewicz

---

Subnodes: Uniwersytet Warszawski (UW)

KEY STAFF: Professors: T. Figiel (30%, IM PAN), St. Kwapień (30%, UW), R. Latała (30%, UW) P. Mankiewicz (50%, IM PAN), A. Pełczyński (30%, IM PAN), M. Wojciechowski (30%, IM PAN). Ass. Professors: K. Oleszkiewicz (30%, UW).

Research interest

The research interest of the Warsaw group concentrates around the following interconnected subjects: probability in $\mathbf{R}^n$, geometry of convex bodies in $\mathbf{R}^n$ and the (local) theory of Banach spaces with emphasis on the geometry of some classical spaces. The probability direction covers such topics as Gaussian measures, random multilinear forms, empirical processes, inequalities and convexity; in the geometry of convex bodies the main subject of study is the geometry and structure of random sections and projections of convex bodies in $\mathbf{R}^n$. The third area of research includes the geometry of spaces of smooth functions and in particular its dependence upon the dimension of the underlying space and the class of derivatives involved.

Principal research personnel

T. Figiel is a well known expert in the area of the local theory of Banach spaces, the co-author of several, now classic papers, e.g. with J. Lindenstraus and V. Milman on measure concentration on Euclidean spheres and Dvoretzky's Theorem.

St. Kwapień is a leading expert in both Probability and Banach space theory. He is the co-author (with W. Woyczyński) of a frequently cited monograph "Random Series and Stochastic Integrals: Single and multiple".

R. Latała was an invited speaker at the International Congress of Mathematicians (ICM) Beijing 2002. In his recent work (with K. Oleszkiewicz) a family of new functional inequalities that interpolates between Poincaré (spectral gap) and logarithmic Sobolev inequalities is established. Also, in another paper, the same authors proved the longstanding conjecture of L. Shepp concerning the behavior of dilatations of convex symmetric sets under Gaussian measures.

P. Mankiewicz works on the asymptotic geometry of convex bodies, in particular on the structure of random sections and projections of convex bodies. In his work (with N. Tomczak-Jaegermann) a detailed description of Banach-Mazur distances between random projections of symmetric convex bodies is given.

A. Pełczyński is an outstanding mathematician whose influence on the development of the theory of Banach spaces is difficult to overestimate. It is enough to mention his contribution in the areas of approximation, bases, operator ideals, local theory and the geometry of function spaces.

M. Wojciechowski exploited the theory of singular integrals to obtain estimates of the growth with the dimension of some analytical characteristics of Sobolev spaces.

K. Oleszkiewicz made his name when jointly with R. Latała, he provided the optimal constant in a Khinchin type inequality for sums with vector coefficients. Some of his other more recent achievements are described above (joint works with Latała).

Mentoring, transfer of knowledge, facilities and infrastructure

The Warsaw node has a very strong tradition in education on both PhD and postdoctoral level (all of its members got their degrees in Warsaw). It organizes several weekly research seminars and has a tradition of hosting young researchers for several months. At present there are 6 PhD students. The facilities in IMPAN include one of the best mathematical libraries in Europe, a state of the art computer lab and a conference center adequate to organize workshops and schools. The team members hold 3 editorships of international journals between them.

Two significant publications:

1. R. Latała: On some inequalities for Gaussiam measures, Procedings of the International Congress of Mathematicians, vol II, 813-822, Higher Education Press, Beijing 2002.
2. P. Mankiewicz & N. Tomczak-Jaegermann: Geometry of Families of Random Projections of Symmetric Convex Bodies, GAFA Geometric and Functional Analysis, 11 (2001) 1282-1326.

TEAM 7 **University of Zaragoza (UZ)** **coordinator: J. Bastero**

Subnodes: Universities Complutense de Madrid (UCM), Murcia (UM), Rey Juan Carlos I de Madrid (URJC), Sevilla (US) and Valencia (UV).
KEY STAFF: J. Bastero (40%, UZ), J. Bernués (30%, UZ), A. García-Olaverri (30%, UZ), J. Tejel (30%, UZ), F. Cobos (30%, UCM), F. Hernández (30%, UCM), A. Suárez (30%, UCM), J. C. García-Vázquez (30%, US), L. Rodriguez-Piazza (30%, US), R. Villa (30%, US), O. Blasco (30%, UV). Ass. Professors: M. Hernández (UM), M. Romance (URJC, UZ).

Research interest

The Spanish team contributes to the project in several areas: asymptotic geometric analysis (UZ, UCM, URJC, US, UV), classical convexity and geometric inequalities (UZ, UM, URJC) and asymptotic combinatorics (UZ). Our group has strong links with the teams at Marne-la-Vallée, Kiel, London, Florence, Freiburg, Vienna, Crete and Tel Aviv. The corresponding subnodes habitually work in research projects supported by the Spanish Agency, which have been steadily renewed for many years. Some of them have participated in the Picasso Programme in cooperation with the Marne-la-Vallée team.

Principal research personnel

J. Bastero is expert in convex geometry and interpolation of operators theory. With **J. Bernués** he extended V. Milman's (from team 12) reverse Brunn-Minkowski inequality to general bodies. With **M. Romance** he solved, using variational methods, several extremal problems associated to dual mixed volumes, in terms of isotropic measures and studied the corresponding reverse inequalities.

A. García Olaverri and J. Tejel research in asymptotic combinatorics, specially on enumeration problems related to geometric structures and on low-complexity algorithms for geometric graphs.

F. Cobos is one of the world leaders in the interpolation theory of operators. Specially relevant are his contributions to the problem of the interpolation of compact operators. Recently he obtained qualitative and quantitative results closely connected to the work of members of team 3.

F. Hernández has made significant contributions to Classical Banach Spaces Theory, in particular on lattice and symmetric structures of Banach spaces, connected with the work of members teams 1 and 3. **A. Suarez** is also involved in the geometry of infinite dimensional Banach spaces

M. Hernández works in classical convexity with links with teams 8, 9 and 11. She studied optimization problems for convex bodies, obtaining inequalities relating geometric magnitudes.

L. Rodriguez-Piazza researches in operator space ideals, vector valued measures and harmonic analysis, especially in topics related to Sidon sets, whose connection to the geometry of Banach spaces was originated by Pisier from team 1.

R. Villa works in the local theory of Banach spaces and especially in the concentration of measure phenomenon on which he collaborated with K. Ball from team 2.

J. C García-Vázquez also researches in the local theory of Banach spaces.

O. Blasco is an expert in harmonic analysis and the geometry of Banach spaces and he has been working in harmonic analysis of vector valued functions. He has succeeded in connecting local properties of Banach spaces and inequalities for vector valued analytic functions in Hardy spaces.

Mentoring, transfer of knowledge, facilities and infrastructure

The subnodes have enough experience in training early-stage researchers and several new Ph.D.'s were obtained advised by members in the subnodes. Effective transfer of knowledge is ensured since the members of the team regularly attend conferences on the topics in the proposal. Moreover, researchers in the subnodes have held long term research stays in other participant institutions of the network as well as receiving visitors that participate in research activities (seminars, lectures and the like) very often. Facilities and infrastructure are provided by the Universities involved. The team members hold 4 editorships of international journals between them.

Two significant publications:

1. J. Bastero, M. Romance: Positions of convex bodies associated to extremal problems and isotropic measures (to appear in Adv. Math.)
2. J. Arias de Reyna, K. Ball, R. Villa: Concentration of the distance in finite dimensional normed spaces. Mathematika, **45** (1998), 245–252.

TEAM 8 | **University of Florence (UNIFI)** | **coordinator: A. Colesanti**

Subnodes: Istituto per le Applicazioni del Calcolo - Consiglio Nazionale delle Ricerche (IAC); University of Modena (UNIMO); University "Cattolica", Milan (UNICA); Polytechnic University, Milan (POLYMI); University of Trieste (UNITR).

KEY STAFF. Professors: G. Bianchi (30%, UNIFI), S. Campi (40%, UNIMO), A. Colesanti (50%, UNIFI), M. Longinetti (30%, UNIFI), G. Talenti (30%, UNIFI), A. Volčič (30%, UNITR). Ass. Professors: P. Dulio (30%, POLYMI), P. Gronchi (30% IAC), C. Peri (30%, UNICA), P. Salani (30%, UNIFI).

Research interest

Geometric inequalities. Analytic aspects of convex geometry. Partial differential equations of elliptic type. Calculus of variations.

Principal research personnel

G. Bianchi. Semilinear partial differential equations of elliptic type: existence and non-existence of solutions, symmetry and geometric properties. Convexity: inverse problems like continuous or discrete tomography and the covariogram problem.

S. Campi - P. Gronchi. Theory of convex bodies; extremal problems in special classes of convex bodies. Continuous transformations of convex bodies and applications to geometric functionals. Inequalities in convex geometry. Geometric tomography: recovery of geometric invariants of convex bodies from data on sections and/or projections.

A. Colesanti. Partial differential equations of elliptic type: geometric properties of the solutions. Theory of convex bodies: extremal problems in special classes of convex bodies, determined by constraints on sections or projections. Brunn-Minkowski type inequalities for functionals in the calculus of variations.

P. Dulio - C. Peri. Inequalities of isoperimetric type: relations between continuous functionals of convex bodies with particular attention to relative isoperimetric inequalities. Geometric tomography: the problem of determining convex bodies or star-shaped bodies through X-rays, in Euclidean and non-Euclidean spaces.

M. Longinetti. Analytic and geometric properties of real-valued convex and quasi-convex functions. Isoperimetric inequalities for curvatures and Minkowski functionals of level sets of solutions to elliptic partial differential equations. Convex tomography for convex planar sets.

P. Salani. Connections between convexity and elliptic partial differential equations. In particular: qualitative properties of solutions to elliptic problems inherited by the domain, like convexity or convexity of level sets; Monge-Ampère equations.

G. Talenti. Partial differential equations of elliptic type; in particular: applications of symmetrization techniques and estimates for the solutions. Calculus of variations, isoperimetric problems. Inverse problems.

A. Volčič. Convex and star-shaped tomography, more specifically the Hammer X-ray problem (its well-posedness, reconstruction algorithms, genericity results) and the covariogram problem. Measure theory.

Mentoring, transfer of knowledge, facilities and infrastructure

The team has currently 4 PhD students. The team organizes a regular seminar in Florence on the themes of analytic aspects of convex geometry and partial differential equations; moreover, a workshop is organised each year in Florence. S. Campi and A. Volčič, together with R. Schneider from team 11 were the organizers of the three international meetings on "Convex geometry - Analytic aspects" (Cortona - Italy, 1995, 1999 and 2003). The team holds 4 memberships of editorial boards of international mathematics journals.

Two significant recent publications

1. S. Campi and P. Gronchi: The L_p-Busemann-Petty centroid inequality, Advances in Mathematics 167 (2002), 128-141.
2. A. Colesanti and P. Salani: The Brunn-Minkowski inequality for the p-capacity of convex bodies, to appear on Mathematische Annalen.

TEAM 9 University of Technology Vienna (TU Wien) coordinator: P.M. Gruber

---

Subnodes: University of Magdeburg (UM), University of Salzburg (US).

KEY STAFF: Professors: C. Buchta (30%, US), P.M. Gruber (50%, TU Wien), M. Henk (30%, UM), M. Ludwig (50%, TU Wien), M. Reitzner (50%, TU Wien) Ass. Professors: A. Schürmann (30%, UM)

Research interests

The Austrian team has a wide range of expertise concerning the project. The main field of research is discrete and convex geometry. The Austrian team has been heavily involved in asymptotic best approximation and asymptotic random approximation of convex bodies, characterization theorems for valuations and quantization problems including applications to areas such as numerical integration and data transmission. For many years, there have been close contacts with other teams, in particular to Freiburg, London, and Budapest.

Principal research personnel

C. Buchta is one of the leading experts on random polytopes. Particularly interesting are his precise results on the determination of various approximation constants. His work is closely related to work of Bárány and Larman, Schneider, and Schütt and Werner.

P. M. Gruber has done pioneering work on asymptotic best approximation of convex bodies by polytopes, on optimal distribution of points on Riemannian manifolds and on the related quantization problem. Besides relations to data transmission, these problems are related to numerical integration, approximation of probability measures by discrete measures, approximation of functions by step functions, and isoperimetric problems. Peter Gruber is one of the editors of the "Handbook of Convex Geometry" and authored with G. Lekkerkerker the standard treatise "Geometry of Numbers". He is a Full Member of the Austrian Academy of Sciences and a Foreign Member of the Russian Academy of Sciences.

M. Henk did fundamental research in discrete geometry and geometry of numbers. Particularly interesting are his solution (with co-authors) of the sausage conjecture of Fejes Tóth for high dimensions and the specification of an effective algorithm to determine the densest lattice packings of convex polytopes in 3-space. He was recently appointed to a full professorship in Magdeburg.

M. Ludwig has done fundamental work on the characterization of valuations on convex bodies. In a joint work with Reitzner she obtained a characterization of affine surface area. In an earlier work she solved the problem of asymptotic best approximation of convex bodies by general polytopes.

M. Reitzner has made deep contributions to the approximation of convex bodies by random polytopes. A major result in this area is his asymptotic expansion for random approximation with respect to quermassintegrals. This essentially generalizes the case of approximation with respect to the mean width dealt with earlier by Gruber and for the planar case by Schneider and Wieacker.

Mentoring, transfer of knowledge, facilities and infrastructure

The team has had a successful record of PhD students. It organises a regular weekly seminar and every year at least one workshop in Vienna. The team members hold 1 editorship of international journals between them.

Two significant publications:

1. P. Gruber, Optimum quantization and its applications, Adv. Math., to appear.
2. M. Ludwig and M. Reitzner, A characterization of affine surface area, Adv. Math. 147 (1999) 138-172.

TEAM 10 **University of Toulouse (UPS)** **coordinator: M. Ledoux**

KEY STAFF: Professors: D. Bakry (30%, UPS), F. Barthe (30%, UPS), Y. Brenier (25%, CNRS, Nice), M. Ledoux (50%, UPS). Ass. Professors: M. Capitaine (30%, CNRS, UPS), M. Casalis (30%, UPS), Th. Delmotte (30%, UPS), A. Guionnet (25%, CNRS, ENS Lyon), X.-D. Li (30%, UPS), L. Miclo (30%, CNRS, UPS), Z. Qian (30%, CNRS, UPS).

Research interests

The Toulouse team has a strong expertise in the analytic and geometric aspects of Markov and diffusion operators, in connection with geometric and functional inequalities on Euclidean and Riemannian manifolds (Sobolev and logarithmic Sobolev type inequalities, isoperimetric bounds, measure concentration). Recent interests also concern statistical mechanics, rough paths, fast decays of nonlinear PDE's as well as random matrices and free probability, in which a number of fundamental tools from the project have been shown to be of central importance. It has a number of cooperations with the teams from Paris and London, as well as worldwide collaborations. It employs 9 researchers. Associated to the team, Y. Brenier (Nice) and A. Guionnet (Lyon) complete the scope of interests covering aspects in mathematical physics and large deviation principles in classical and free probability theory.

Principal research personnel

D. Bakry is a world-renowned expert on the interactions between functional analysis, geometry and probability theory. He initiated the use of geometric tools, such as Riemannian curvature and dimension, in the study of functional inequalities of Sobolev and logarithmic Sobolev type for abstract Markov operators. Together with M. Ledoux, he established the infinite dimensional version of the Lévy-Gromov isoperimetric comparison theorem. He played a founding role in the development of the Toulouse probability group, and supervised a number of PhD students.

F. Barthe has pioneered the use of optimal transport to derive functional geometric inequalities. He was recently awarded the Peccot lecture at Collège de France. Jointly with Artstein (team 12), Ball (team 3) and Naor they solved an old conjecture by Shannon concerning entropy increase in the Central Limit Theorem. He will move next year from team 1 to Toulouse. Fruitful interactions in the framework of the project are expected.

Y. Brenier is the world leader in optimal mass transportation, Monge-Kantorovitch theory and geometric PDE's related to hydrodynamics and electrodynamics. In the early 90's, he established the famous mass transportation by the gradient of a convex function, that is now at the basis of numerous analytic, geometric and probabilistic developments at the heart of the project. His broad interests cover a number of areas in mathematical physics. He educated several already famous PhD students. He was invited speaker at the Beijing 2002 World Congress.

M. Ledoux worked with M. Talagrand (team 1) on the isoperimetric aspects of probability in Banach spaces. He further developed the functional tools for the concentration of measure phenomenon, presented in a recent book at the AMS. His recent interests are concerned with applications of the method from high-dimensional phenomena to random matrices and random growth models.

A group of brilliant young permanent researchers includes **M. Capitaine, M. Casalis, Th. Delmotte, X.-D. Li, L. Miclo and Z. Qian**. Their interests cover random matrices and free probability, analysis on path spaces and rough paths, and Markov kernels on graphs with applications to statisitical mechanics. **A. Guionnet**, honoured by international prizes, is one of the leading young probabilists, specialist in the theory of spin glasses, asymptotics of matrix integrals and free probability.

Mentoring, transfer of knowledge and infrastructure

The University of Toulouse has an excellent record of successful PhD students. Recently, a group of PhD students of D. Bakry and M. Ledoux wrote a book on logarithmic Sobolev inequalities, published by the Société Mathématique de France. The team has currently 6 PhD students (not counting the PhD students of Y. Brenier and A. Guionnet, 3 at the moment). Regular seminars and workshops are organised. The Laboratoire de Statistique et Probabilités, of which the principal investigators are members, is one of the central places for Probability Theory in France. The international meeting, “Journées de Probabilités”, regularly takes place in Toulouse and gathered this year more than 100 participants. The University of Toulouse is one of the broadest universities in France, and offers all the necessary facilities for research and teaching. The team holds 8 memberships of editorial boards of international mathematics journals and 2 managing editorships of such journals.

Two significant publications:

1. D. Bakry, Z. Qian, Some new results on eigenvectors via dimension, diameter and Ricci curvature, Adv. in Math. 155 (2000), 98-153.
2. S. Bobkov, I. Gentil, M. Ledoux, Hypercontractivity of Hamilton-Jacobi equations, J. Math. Pures Appl. 80 (2001), 669-696.

TEAM 11 **University of Freiburg (UF)** **coordinator: R. Schneider**

Subnodes: University of Karlsruhe (UK), Technical University of Munich (TUM).

KEY STAFF: Professors: P. Gritzmann (25%, TUM), G. Last (30%, UK), R. Schneider (50%, UF), W. Weil (30%, UK). Ass. Professors: A. Brieden (30%, TUM), D. Hug (30%, UF), M. Kiderlen (30%, UK).

Research interest

The Freiburg team has a wide-ranging expertise in convex geometry and its applications. The main research interests of the team members in Freiburg and Karlsruhe cover classical convexity, integral geometry and stochastic geometry. The Munich subnode has a long record in studying algorithmic problems that can be characterized by the underlying interaction of Graph Theory, Mathematical Programming, Computational Complexity and Computational Convexity; applications currently studied include some in agriculture, others in medicine and yet others in materials science. There exist various cooperations with the team in Budapest, on the algorithmic theory of convex bodies, especially on optimal approximations of high-dimensional spheres by polytopes, and with the teams in Florence and Vienna.

Principal research personnel

A. Brieden is an experienced researcher in Discrete Mathematics, Combinatorial Optimization and Algorithmic Geometry. He has recently finished his Habilitation thesis on some basic problems in Computational Convexity and applications to the consolidation of farmland.

P. Gritzmann is Head of the Munich research group in Discrete Mathematics, Combinatorial Optimization and Algorithmic Geometry. His work ranges from theoretical geometric research to the development of algorithms for real world tasks. Together with V. Klee, University of Washington, he has founded the research field of Computational Convexity.

D. Hug is working on different aspects of curvature and surface area measures in Euclidean and Minkowski spaces. A particular feature of this work is the use of geometric measure theory and results from non-smooth analysis.

M. Kiderlen is studying inverse (reconstruction) problems related to convex and stochastic geometry, including complexity issues. His research is also concerned with uniqueness questions related to averages of intersections or projections of convex bodies.

G. Last is a well-known specialist in random point processes. He works on stochastic processes and their application in finance, queuing and reliability. In recent years he became active in stochastic geometry related to convexity. In a series of papers, he developed and applied generalized contact distributions and contributed to the theory of curvature measures.

R. Schneider is one of the leading experts in convex geometry and its applications. His recent research in convexity is concerned with projection functions, mixed functionals, valuations, and extremal problems. A main theme of his work is the application of results from the geometry of convex bodies to other fields, in particular extremal problems in stochastic geometry or, more recently, integral geometry in Minkowski and Finsler spaces.

W. Weil is an expert in geometric integral transforms, inverse problems, zonoids, and functional analytic methods in convexity. He has played a major role in the development of translative integral geometry, mainly in view of applications to stochastic geometry. His work on stochastic geometry is widely known. Some of his recent investigations are devoted to generalized contact distributions, distance measurements and functional densities of random closed sets.

Mentoring, knowledge transfer, facilities and infrastructure

The Freiburg team has a long experience in training and mentoring young researchers. This is documented by various books and lecture notes, written by its senior members, which grew out of class notes and some of which have immediately become standard references in the field. Members of the team have, up to now, organised 20 Oberwolfach conferences. Currently, there are 9 PhD students included in the team. There are an excellent library and up-to-date computer facilities available at the Freiburg node. The team members hold 3 editorships of international journals between them.

Two significant publications:

1. D. Hug, M. Reitzner and R. Schneider, The limit shape of the zero cell in a stationary Poisson hyperplane tessellation, 29 pp. *Ann. Probab.* (in print);
2. W. Weil, Mixed measures and functionals of translative integral geometry. *Math. Nachr.* 223 (2001), 161-184.

TEAM 12 **University of Tel Aviv (TAU)** **coordinator: V. Milman**

Subnodes: Technion, University of Haifa (UH), Weizmann Institute (WI).

KEY STAFF: N. Alon (30%, TAU), I. Benyamini (30%, WI), Y. Benyamini (30%, Technion), E. Gluskin (30%, TAU), Y. Gordon (40%, Technion), M. Krivelevich (30%, TAU), V. Milman (50%, TAU), A. Pinkus (30%, Technion), Sh. Reisner (30%, UH), Sh. Safra (30%, TAU), G. Schechtman (40%, WI), M. Sodin (25%, TAU), B. Tsirelson (30%, TAU), O. Zeitouni(25%, Technion).

Research interests

The Israel team represents very broad directions of expertise in most high dimensional theories. The scientific level of the team may be judged by the fact that Alon was a Plenary Invited speaker at the 2002 ICM, Alon and Milman were Plenary Invited speakers at the 1996 European Congress, and also 6 invited sectional talks were delivered by team members at different ICMs.

Principal research personnel

N. Alon has been one of the most prolific and influential figures in Combinatorics during the last decade. He contributed enormously to almost all areas of Combinatorics, and especially to those in Asymptotic Combinatorics. His more than 300 articles in these areas have revolutionised the whole field of discrete mathematics bringing to it new structures and approaches, using methods from Probability, Algebra, Topology and Functional Analysis.

I. Benyamini, works on the connections between randomness and geometry. He was one of the group that developed Probability on trees, and later moved to study Probabilistic aspects of models arising from statistical physics such as percolation, first passage percolation and uniform spanning trees. He is also involved in algorithmic and computational applications of random walks and related processes.

Y. Benyamini, after impressive works on the factorization of operators, turned to non-linear theory (see his joint book with Lindenstrauss on "Geometric Nonlinear Functional Analysis, Colloquium Publications, AMS, vol 48, 2000).

E. Gluskin started the theory of random spaces called "Gluskin spaces" and continued this development.

Y. Gordon pioneered the development of the theory of operator ideal norms for the study of various geometric parameters of normed spaces. He also discovered min-max probabilistic principles for Gaussian processes.

M. Krivelevich is one of the strongest researchers in the young generation of combinatoricists. His recent research was concentrated around topics such as the asymptotic behavior of eigenvalues of random graphs and matrices, algorithmic applications of spectrum-related parameters, models and asymptotic properties of various random structures.

V. Milman initiated the development of the Concentration Phenomenon and its use in Geometry. Jointly with Gromov (team 1) he connected it with topology and properties of eigenvalues of Laplacian Operators. Later Maurey (team 1) and Schechtman introduced martingale methods in their study of Concentration and Talagrand (team 1) and Ledoux (team 10) developed Concentration ideas in Probability with an impressive row of applications. Milman also pioneered the Asymptotic Theory of Convexity. In recent joint works with Schechtman (Duke J., (1997) Math. Annalen (1998)) they discovered the first examples of phase transition type phenomena in this theory.

A. Pinkus, a well known expert in Approximation Theory, has recently shifted his activity into "Learning Theory" which is now using the tools of the Asymptotic Theory of Normed Spaces.

S. Reisner, besides fundamental results in Convexity, is now developing theoretical and constructive approximation theory for polytopes and general convex bodies.

S. Safra is an outstanding expert in Complexity theory. He is actively using many aspects of Asymptotic theory in his work. He treats Boolean functions as functions on the high dimensional cube, analyzing their Fourier transforms, and proves concentration of measure which plays an important role in various aspects of complexity analysis. He also studies and discovers different high dimensional phenomena (such as Threshold and Phase transitions) in Complexity Theory.

G. Schechtman is one of the coauthors of the deepest study of the linear structure of ℓ_p^n - spaces.

M. Sodin, an expert in Classical Analysis, recently applied his expertise to achieve significant progress in a number of problems of Asymptotic Convexity. He (jointly with Nazarov and Volberg) found a geometric version of a result of Kannan-Lovász-Simonovich (team 4) and applied it to obtain sharp estimates of volumes of sub- and super-level sets of polynomials and analytic functions. Last year he, jointly with Tsirelson, studied Gaussian random analytic functions and proved asymptotic normality for smooth functionals of the set of zeros. They are continuing this study and are connecting it with transportation of measure.

B. Tsirelson, a leading expert in probability and one of the discoverers of the isoperimetric principle for Gaussian multidimensional variables, investigates product systems, shedding new light on highly nonlinear functions of many independent random variables, scaling limits and stability. He also applies measure concentration in the theory of large groups of measure preserving transformations.

O. Zeitouni, another world expert in Probability has his recent works centered around the study of spectral properties of large random matrices. He is actively using stochastic analysis, free probability and large deviation techniques.

Mentoring, transfer of knowledge, facilities and infrastructure

The team has a large experience in the training and mentoring of young researchers. At present, it has 25 graduate students working under the supervision of the senior team members in all directions of the team's expertise. The publication records of these students are excellent with many papers published and submitted to Annals, Inventiones, GAFA and other top journals. There are 8 weekly seminars in all 3 subnodes, and many special courses are given regularly. On average once every 3-4 weeks the team holds a GAFA seminar which brings together all researchers and students in these directions. A regular stream of visitors from other teams share their expertise at this world famous seminar. The infrastructure, library and facilities available to young researchers in all subnodes are excellent. The team members hold 26 editorships of international journals between them, 3 of which are managing editorships.

Two significant publications:

1. N. Alon, Discrete Mathematics: methods and challenges, Proc. of the International Congress of Mathematicians (ICM), Beijing 2002, China, Higher Education Press vol 1 (2003), 119-135.
2. B. Klartag, V. Milman, Isomorphic Steiner Symmetrization, Inventiones vol 153:3 (2003), 465-485.

TEAM 13 Mathematical Division of the Institute for Low Temperature Physics, Ac. Sci. of Ukraine, Kharkov, Ukraine (MDILTP) coordinator: L. Pastur

---

Subnode: Kharkov State University (KSU).

KEY STAFF: Professors: I. Chueshov (30%, KSU), L. Pastur (50% , MDILTP) M. Shcherbina (female, 30%, MDILTP) Ass. Professors: A. Rezounenko (30%, KSU), V. Vasilchuk (30%, MDILTP),

Research interest

The expertise of the team covers several fields of mathematical physics: the spectral theory of random operators and matrices, statistical physics of disordered systems, theory of infinite-dimensional dynamical systems and their applications (condensed matter, wave propagation, elasticity and hydrodynamics). The group pioneered studies of universality in random matrix theory and of self-averaging and phase transitions in the rigorous statistical mechanics of disordered systems and certain aspects of dynamical theory of the PDE's. All these fields include a strong high-dimensional component, requiring various methods of asymptotic analysis, in both its geometric and analytic aspects.

Principal research personnel

L. Pastur is one of the world-wide leaders in spectral theory and the mathematical physics of disordered systems. He is a full member of the Academy of Sciences of Ukraine, coauthor of two well known books (1988 and 1992) and of a number of reviews (the last one in the Lectures Notes in Mathematics, **1801** (2003). He will be one of the leaders of research and training related to task 5, but also will develop a collaboration with experts from teams 2, 4, and 12, on the asymptotic analysis and threshold phenomena (tasks 1, 3 and 6).

M. Shcherbina works actively on random matrices and the theory of spin glasses and neural networks. Her recent bounds on the capacity of neural networks and a complete solution of the Gardner problem is one of the remarkable events of the field. She is invited speaker on the next European Congress of Mathematicians (Stockholm, 2004).

I. Chueshov is a well-known expert in infinite-dimensional dynamical systems. He has published more than 60 scientific papers, including two books. Most of his research deals with a non-regular and long-time behavior of nonlinear PDE's of physics and mechanics, in particular with the geometric and analytic structure of attractors.

A. Rezounenko made serious contributions to the theory of retarded evolutionary partial differential equations (attractors and inertial manifolds) with applications to non-linear oscillation problems.

V. Vasilchuk has important results in the eigenvalue distributions of several ensembles of random matrices and their links with the free probability studies of recent years.

Mentoring, transfer of knowledge, facilities and infrastructure

The team consists of members of two most significant mathematical units in Kharkov, one of the traditional centres of science and high level technology in Eastern Europe. The MDILTP is a research institution, but it has the PHD school. The KSU is one of the oldest Universities of the country. There are currently 5 PhD students. The members of the MDILTP group are involved in the part-teaching at KSU, and all members of the team participate in the Kharkov seminar in mathematical physics. The team regularly participates in organizations of international conferences in Kharkov, partly supported by the INTAS and the NATO. The team has well-established links with the both French, and the Israeli teams, and also with research groups in Europe, US, and Canada. The team members hold 3 editorships of international journals. The team has also a long tradition of interaction with the theoretical physics and applied mathematics communities. There is a necessary infrastructure, a library and computer facilities.

Two significant publications:

1. M. Shcherbina, B. Tirozzi: Rigorous solution of the Gardner problem, Commun. Math. Phys. 234 (2003), 383-422.
2. I. Chueshov: Monotone random systems. Theory and Application, (Lecture Notes in Mathematics, 1779) Springer, Berlin-Heidelberg-New York.

B3.2. Intensity and quality of networking

The proposed network has 9 teams from EU member states, 3 teams in Associated Countries (Poland, Hungary and Israel) and 1 team from a third country. The less-favoured regions impacted by the programme are Crete, Athens and Samos which are the locations of the Greek node and subnodes and Andalucia and Murcia which are subnodes of the Spanish node.

Most research in mathematics is conducted by individuals or by small groups that form spontaneously (at conferences or as a result of extended visits): it is rarely possible to divide research problems into parts that can be allocated to separate groups. Thus, where research is concerned, the major effects of the network will be to provide opportunities for contact, which will then be acted upon by individuals.

While the most important contributions to training will come from the visits of younger researchers and the summer schools described in section 2, research collaboration will also be fostered by the organisation of research conferences, (at least 1 per year), and by secondments of senior personnel to other sites (in particular to the Spanish and Italian sites).

It is possible to identify several research tasks that will be undertaken jointly by teams from different nodes because there are existing collaborations detailed in section 3.3. New collaborations that can be foreseen are those between the French and Italian teams (on geometric/functional inequalities), the contacts of British and French team members with the Kharkov team (eigenvalues of random matrices) and the potentially huge expansion of the links between the combinatorial groups in Hungary, Britain and Israel and the more "geometric" groups at many nodes.

In view of the fact that mathematicians from different disciplines are involved in this project, the ability of the different groups to talk to one another is vital. This has been amply demonstrated by a number of recent programs and meetings around the world. The most obvious one was the 2-month programme at PIMS in Vancouver which took place last year. This programme brought together researchers in all the main areas of the project and the high quality of the lectures and dedication of the senior personnel made it possible for everyone (especially students and postdocs) to grasp all the material. The researchers in all fields of this programme have a tradition of writing their research articles and monographs for as broad an audience as possible. The expository lectures at summer workshops will be the principal means by which the network itself will enable its members to overcome disciplinary boundaries.

The candidate countries, Poland and Hungary, that are contributing teams to this proposal, present no special problems of integration. Hungary is one of the most prolific sources of top-level scientists in combinatorics and the researchers in both countries have already collaborated on research articles with members of many other teams. Special attention will be paid to the teams from Spain and Italy which have fewer senior personnel. The teams from Spain and Greece have nodes or subnodes located in less-favoured regions. Funds will be made available for senior scientists from other parts of the network to visit these sites for several months at a time. The Greek team is in an especially good position to benefit from the network: almost all the team members are young and many of the mathematical directions of the programme have recently taken root in Greece.

The integration of the team from Kharkov will be led by teams 1 and 10 from France and team 12 from Israel, which already have strong links with several members of the Kharkov team. The team from Israel is very much in the position of the Polish and Hungarian teams: its members already travel widely in Europe and collaborate with members of other nodes.

B3.3. Relevance of Partnership Composition

The introduction to section 3.1 gave an overview of historical links between teams in the context of complementarity. Within the subsections detailing the expertise of individual teams are listed several bilateral programmes such as an Alliance/British Council grant (UMLV/UCL), a Picasso programme (UMLV/UZ), and a German/Israeli programme (UKiel/TAU). Many team members have visited other nodes to give seminars and lecture series.

Recently members of different teams have collaborated in the organisation of research and training conferences (some of which are mentioned in section 4.3), for example, Milman, Schechtman and Tomczak-Jaegermann from Canada organised a 3-month programme at PIMS in Vancouver in 2002, Carbery, Gowers and Milman organised an ICMS conference in Edinburgh in 2002, Ball, Milman, Pajor and Lovász organised the 6-month programme and its workshops at MSRI in 1996. Starting in the academic year 2002 the Seville subnode has run an analysis programme consisting of a series of lectures on topics covered by this proposal: speakers have included Pajor, Ball, Hinrichs, Pełczynski and Gordon from the network.

In this section we mention some of the many specific collaborations that have taken place between members of different teams and that have resulted in joint articles. The teams from Marne la Vallée, London, Kiel, Budapest, Heraklion and Tel Aviv have engaged in numerous collaborations in different fields: for example there are joint articles by

Ball/Barthe, Gowers/Maurey, Schütt/Gordon/Werner/Reisner, Milman/König, Alon/Lovász, Welsh /Dyer/McDiarmid/Simonovits, Giannopoulos/Milman, Fradelizi/Giannopoulos/Meyer, Larman / Dalla, Carl/Pajor, Gromov/Milman, Preiss/Schechtman.

The teams from Warsaw, Zaragoza, Florence, Vienna, Toulouse, Freiburg and Kharkov each have published links with 1 or 2 other nodes. One of the aims of the network is that these nodes will become more closely linked to all the others. Examples of existing collaborations involving these groups are:

Schütt/Kwapien, Ball/Villa, Ledoux/Talagrand, Brieden/Gritzmann/Lovász/Simonovits, Campi/ Weil, Bianchi/Gruber and Hug/Reitzner/Schneider.

The crucial way in which the network will build new collaborations is by bringing together mathematicians from different fields who have only recently begun to meet at conferences and workshops. Some specific collaborations are predicted in section 3.2 above. Just as teams from different countries already collaborate within particular subfields, it will be important for the growth of *cross-disciplinary* collaboration that several teams include researchers from a variety of fields who know one another by belonging to the same university.

The team based in Kharkov in the Ukraine is outside the EU and associated states. This team is included because of its extremely strong research record in random matrix and spin glass theories (and other parts of mathematical physics): an area which is increasingly finding applications in asymptotic combinatorics and geometry as well as physics. It is also the principal research topic for several students in other parts of the network, in particular those working with Szarek from the Paris node and Ledoux from the Toulouse node. It is highly desirable that the exciting developments in this field should be disseminated widely around the network. Moreover, members of the Kharkhov team already have active research contacts with members of the other nodes, in particular 1, 10 and 12.

B4 MANAGEMENT AND FEASIBILITY

B4.1. Proposed management and organisational structure

The training and research activity will be organised on both network wide levels and local levels.

Network wide activities and management

The main activities are

a) annual one-week conferences, with two or three series of mini-courses on specific subjects in the morning and short talks and poster sessions on all topics relevant to the network workplan in the afternoons;
b) specialised workshops and summer schools;
c) annual three day workshops with one hour lectures given by all the early-stage or experienced mathematicians recruited by the network;
d) the maintenance of a network information centre including a web homepage and a preprint database and copies of selected lectures given by network members.

This part of the organisation and management will be carried out by the **Network Coordinator**. He will also collect at the end of each year scientific and administrative information from the local coordinators and task coordinators (see below), control preparation and submission of annual and mid-term reports, participate in the work of the Steering and Advisory Committees (see below), visit teams, and will be in charge of the financial management of the network, to be described below.

The amount of work that is necessary to run and to control a training and scientific activity of the level and range of this programme will certainly require additional structures.

It is assumed first that the administrative staff of the Mathematics Department of the University Marne-la-Vallée (including a software engineer) will provide appropriate support for the organisation and management activities, including that of the Network Coordinator.

We plan also that local coordinators and leaders of subnodes, in addition to carrying out their local functions, will form the **Steering Committee** of the network. The Committee will meet at least once a year and the Network Coordinator will act as its chairman. It will choose or approve locations and large scale topics of conferences, workshops, and summer schools, ratify appointments made by the various nodes, and approve the annual and mid-terms reports and allocation of funds for the periodic payments.

As well as the Steering Committee, the Network will have a Scientific Committee, which will supervise the scientific content of the network's activities. This Committee will make suggestions to the Steering Committee for programs and topics for conferences, workshops, and schools (including themes for the mini-courses and speakers). It will identify relevant conferences outside the Network, at which the network should be represented.

For each of the six main tasks of the Network workplan two or three teams will be in charge of the management of research, training and transfer of knowledge activities (see the details in the table given in B1.5 after the description of the network workplan) and will designate every year a **Task Coordinator** for the task. S/he will be responsible for the selection of series of short talks in this direction of research for the annual conference and the organisation of mini-courses at the annual conference. This responsibility includes the implementation of a relevant bibliography on the network homepage before the annual conference and, whenever possible, the organisation of the publication (as survey article or lecture notes) of these mini-courses.

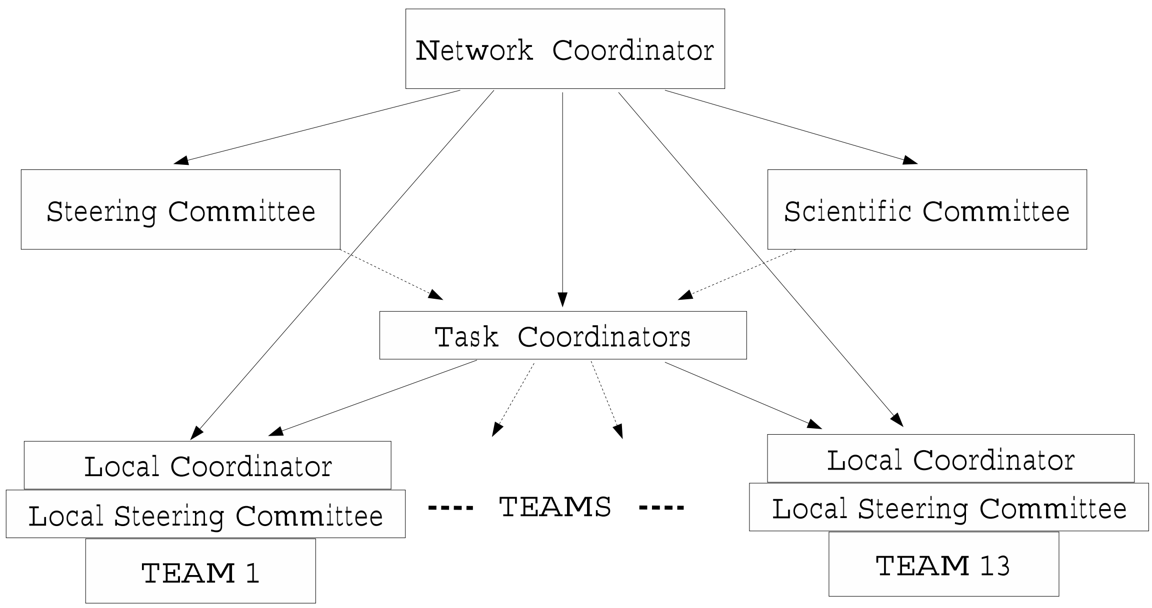
A (female) mathematician will be appointed to monitor the gender issue, to diffuse information about the policy of the European Commission on the inclusion of women, and to inform associations of women in mathematics about the network activity and to inform the network about the activities of these associations. She will sit on the steering committee so as to provide input into the decisions concerning the allocation of visiting positions for young and more experienced researchers.

Local activities and management

The main activities are

a) training-through-research under supervision of experienced researchers;
b) structured training courses on the topics that are best represented in the team;
c) establishing and carrying out of Personal Career Plans for the members of the team;
d) regular seminars to inform members of the team and visitors about recent results in the field and working groups to make detailed studies of recent results of primary interest for the team;
e) provision of mutual visits and participation of members of the team in conferences relevant to network objectives;
f) participation in the organisation of scientific and training meetings that take place on the premises of the team.

This part of organisation and management will be carried out by each **Local Coordinator**, who will also be in charge of the financial management of the team. The local coordinator, scientists in charge and experienced researchers, who are permanent members of the respective university will form the **Local Steering Committee** with the local coordinator as chairman. The committee will meet regularly to deal with questions related to the local network activity. In particular, the Committee will choose between competing claims for visiting appointments and assign a network tutor to each early stage researcher in the team. The tutor will supervise the training and research activity of the early stage researcher.



Financial strategy

The funds will be managed centrally, in order to introduce some flexibility into the financial management. We predict that the advance payments will be distributed according to the proportions of the global budget, but also that discrepancies between the appointment schedules of the various teams (unavoidable in order to get a reasonable number of applicants for each position) will force us to adjust periodic payments to the progress of appointments in the various teams. The proportion of the funds retained by the coordinating entity, if any, should not exceed 2%. Concretely the network coordinator will propose to the Steering Committee an allocation of funds for the first and second periodic payments, the initial payment being distributed according to the proportions of the global budget. All decisions concerning management of funds will require agreement from the members of the Steering Committee.

B4.2. Management know-how and experience of network co-ordinator

The network coordinator has long experience of managing a variety of institutions and projects. He was head of the Department of Mathematics at Marne-la-Vallée for 2 years, and has been heavily involved in the education of teachers as "directeur d'études" and head of this branch of education in his university. He is a member of the scientific committee of his university and has played a central role in the mathematical steering of his university. He is currently head of the hiring committee in mathematics and was an "expert evaluator" for research projects, at the Ministry of Education in 1995-1996.

He is one of the organizers of the Analysis seminar in Paris, has wide experience of supervising doctoral studies and he and B. Maurey are the main scientific leaders of the mathematics group at Marne-la-Vallée.

The network coordinator has a global overview of the scientific aspects of the programme through his contacts with mathematicians of all the disciplines represented and has experience of managing bilateral European projects, in particular with the United Kingdom, Spain and Israel.

He has organised or co-organised many workshops and conferences: among them the conference in Marne in 1994 on convexity which was attended by over 200 participants. This conference was historically important, because it was the first large meeting which brought together internationally leading researchers in probability theory, high-dimensional geometry, combinatorics and theoretical computing.

He organised (jointly with L. Lovász) the workshop on random methods in convex geometry at the Mathematical Sciences Research Institute in Berkeley, in 1996; a conference at CIRM in 2000 on isoperimetric inequalities and the transportation of measure; a workshop at the Pacific Institute of Mathematical Sciences in 2002 on convex geometric analysis and a conference in Marne in 2003 on Machine Learning and Asymptotic Analysis. Finally he is one of the organizers of the trimester programme that will be held at the Centre Emile Borel in IHP in the spring of 2006.

B4.3. Management know-how and experience of network teams

1– Paris team. Several members of the team have wide experience in local management: that of the coordinator was mentioned above. Pisier was for many years the Director of the Equipe d'Analyse Fonctionnelle at Paris VI and has organised numerous conferences. Meyer and Szarek were responsible for several exchange programs, with Israel and Poland and Meyer is currently head of the Department of Mathematics at Marne-la-Vallée. Gromov, Maurey, Pisier and Talagrand will play an important role in our scientific committee. The node is coordinating the network and will participate in the organisation of several workshops, in particular of the trimester workshop at Centre Emile Borel (IHP- Paris) in spring 2006 and the one at Warsaw in 2007 already scheduled.

2– London team. Many members of the British team have extensive experience of research organisation and management. Ball was one of the main organizers of the MSRI programme in convex geometric analysis in 1996: a 6 month research programme covering a large subset of the topics of this proposal. Ball also organised a 3-day conference at UCL two years ago and is on the organising committee for the AMS Joint Summer Research Conference on Gaussian measure and geometric convexity scheduled for next summer.

Larman is the head of the mathematics department at UCL as well as being an active research mathematician.

Preiss has been the leader of numerous research projects requiring international cooperation: a Marie Curie training grant being among the most recent. He was one of the driving forces behind the famous winter schools in the former Czechoslovakia, which were one of the few venues which attracted Western mathematicians into the former Eastern bloc. Preiss is also a member of the standing committee of the London Mathematical Society which coordinates a good deal of mathematics research activity in Britain.

Carbery has been extensively involved in the operation of the ICMS: an institution set up with European support to promote the development of mathematics in new directions and to encourage young researchers. The ICMS is based in Edinburgh: Carbery is one of the local members of the executive committee. He himself was the principal organizer of a recent 2-week instructional and training conference in several of the areas of this program: a meeting which many have deemed to be among the most successful ever hosted by the ICMS. As mentioned elsewhere, co-organizers of this meeting were Gowers from the British team and Milman from the Israeli.

Welsh has been chairman or member of organising committees and programme committees for numerous international conferences in Europe, the US and even Australia. He is one of the leading researchers in the important RAND-APX series of programmes funded by the European Union.

The British team will organise a summer school in 2008.

3– Kiel team. The scientist-in-charge of the Kiel team, König, has extensive administrative experience and experience in project management. He has had administered funds from the DFG, DAAD, from GIF and from NATO involving cooperations with Israel, Canada and Spain. He organised international conferences in Kiel in 1998 and 2003, with 240 and 55 participants, respectively. König has been Dean of the Faculty of Science and Mathematics of the University of Kiel from 1998-2000 and hence was involved in many decisions on appointments of personnel. He is also a member of the Beirat of the (joint German-Israeli) Landau Center for Analysis at the Hebrew University of Jerusalem.

Other members of the team have experience in administrative management. Carl has been Dean of Mathematics at the University of Jena.

4– Budapest team. Lovász has wide experience in organising such projects. Sós has a reputation as an excellent organiser: besides being one of the main organisers of a dozen conferences, she was the Hungarian inspiration behind an Austro-Hungarian series of mini-conferences, and later a wider cooperation with the ZIB in Bielefeld (Germany).

Simonovits co-organised several conferences: one of the largest one was the Erdős conference 1999.

Bárány and Füredi have organised several mini-conferences and participated in organising larger conferences as well.

The Hungarian team will organise a winter school in 2007 on "Asymptotic Combinatorics: methods and applications."

5– Heraklion team. Several members of the team are experienced in departmental management. Papadimitrakis is the coordinator of the Greek node in the network "Harmonic Analysis and related problems". Members of the team are experienced in the organisation of national and international conferences. Since 2000, a conference for young researchers working in analysis has been held annually in Heraklion. An international workshop on "Convex Geometric Analysis" was organised by Giannopoulos at Anogia (Crete, August 2001) with co-organisers Milman, Schneider and Szarek from other teams.

6– Warsaw team. Several members of the Warsaw node have experience in the management of the mathematics institute of the Polish Academy of Sciences. Currently, Figiel is the Deputy Director of the Institute and Mankiewicz is the head of PhD programme of the Institute. Pełczyński was head of a joint Polish-French research programme. Several members of the team participated in grants involving international collaboration. All of the members of the team participated in different grants on the national level (KBN) with Pełczyński, Kwapień, Figiel, Mankiewicz heading such grants. Most of the members of the team have been members of Organising Committees of international conferences. Also, the Institute of Mathematics PAN has been involved in several projects sponsored by the European Union. The team will organise a summer school in 2007, "Probabilistic Methods in Geometry."

7– Zaragoza team. Several people in the Spanish group have experience in department management (Bastero, Blasco). Members of several subnodes (Valencia, Zaragoza, Sevilla) are responsible for the organisation of conferences within the EARCO "Encuentros de Análisis Real y Complejo" project, a yearly conference of mathematicians expert in real and complex analysis. The node coordinator has been involved in the organisation of the international conference ("Probability and Banach spaces" held in Zaragoza in 1985), an international colloquium supported by the Picasso programme of cooperation with the French team in 1994 and the "Jornada de Análisis Funcional" held by the RSME in Zaragoza in 2002.

Cobos, Hernández and Suárez organised the "Colloquium del Departamento de Análisis Matemático" in the Universidad Complutense de Madrid for several years, a well established seminar and research publication in which mathematicians from all over the world participate. Also Bastero was the organizer of the weekly "Seminario Rubio de Francia" held in Zaragoza since the last seventies, where research lectures are given by mathematicians from many countries.

Every subnode in the Spanish team has managed research projects funded by Spanish agencies (CAICYT, DGICYT, DGES). All subnodes have experience in the mobility of university students within the Socrates-Erasmus programme and Tempus. The Sevilla subnode is currently managing a Picasso programme with the French team.

8– Florence team. Campi is the team coordinator of a research project on Convexity funded by the INdAM (Istituto Nazionale di Alta Matematica).

Volčič has been the director of the Department for Mathematical Sciences of the Institute for Applied Mathematics at the University of Trieste. He is the local coordinator at Trieste of the national project on Real Analysis and Measure Theory. He administrated grants from the Italian Research Council (CNR). He is a Member of the Scientific Board of Istituto Nazionale di Alta Matematica and of the Italian Mathematical Union. He was President of the Mathematical Committee of the Italian National Research Council (CNR).

Campi and Volčič organised, together with Schneider from team 11, the three international meetings entitled "Convex Geometry - Analytic Aspects" at the meeting center in Cortona, Italy.

Peri is member of the committee which administers the funds for mathematical research at the University "Cattolica", Milan.

The team will organise a school on geometric inequalities in Spring 2005.

9– Vienna team. The team coordinator has expertise in department management (in the division of analysis). He has been the head of the Mathematics division of the University of Technology in Vienna (for 6 years), president of the Austrian Mathematical Society (for 4 years), chairman of the Mathematics committee of the Austrian Academy of Sciences (for 3 years). He had the main responsibilities for the Austrian Mathematical congress in Innsbruck and the Austrian Mathematical meetings in Linz and Leoben. He has been twice co-organiser of the Oberwolfach meeting on Convex Geometry. He organised numerous conferences and workshops on Convex Geometry and related fields.

There will be a workshop in the Schrödinger Institute in Vienna in July 2005 on high-dimensional geometry and convexity.

10– Toulouse team. The Toulouse team was already part of the Stochastic Analysis network (1996-2002), and is well-experienced with the management of such a network (Bakry was the team coordinator). A number of meetings and conferences were organised in Toulouse in the past few years (Journées de Probabilités, Fermat Conference), showing evidence of the experience and ability in this respect. Ledoux is co-organiser of the joint meeting between the Canadian and French Mathematical Societies that will be held in Toulouse in July 2004.

The Laboratoire de Statistique et Probabilités, of which the principal investigators are members, is one central place for Probability Theory in France, and is part of numerous, French and international, research programs. The team is also part of the French GDR (French training network) on interacting particle systems.

The team will host a workshop in January 2005 on random matrices and probability.

11– Freiburg team. Several members of the Freiburg node are experienced in departmental management as well as in the organisation of international scientific conferences. The team coordinator, Schneider, was Head of Department in 1971/72, 1978/79, 1992, "Students' Dean" in 2000, and is presently Head of the Faculty of Mathematics and Physics. He was co-organiser of 10 Oberwolfach conferences on *Convex Bodies*, from 1974 to 1993, and of three international workshops on *Convex Bodies – Analytic Aspects* in Cortona in 1995, 1999, and 2003; he was on the scientific committee of several international conferences.

Weil was chairman of his department 1991 – 1993, and is presently vice chairman (since 1999). He was co-organiser of 5 Oberwolfach conferences on *Stochastic Geometry* (and related topics), from 1983 to 2002.

Last has organised several invited conference sessions as well as workshops on stochastic processes and stochastic geometry. He successfully applied for 3 DFG-grants (1995 – 1996, 1998, 2002 – 2004) and for a research grant of the Volkswagen Foundation (2002).

Gritzmann is currently president of the German Association of Mathematicians (DMV), and was co-chair of the DMV-special interest groups on Geometry and on Discrete Mathematics. He was chair of the Department of Mathematics of the Technische Universität München (1998-2000), is a member of the board of the Deutsches Museum München. Gritzmann is Chair of the graduate programmes (Graduiertenkolleg) on Mathematical Optimization (University of Trier, 1995-1997) and Applied Algorithmic Mathematics (Technische Universität München (since 1998). He has been the organiser or coorganizer of various international meetings on Computational Convexity, Discrete Mathematics, Operations Research and Optimization, and Discrete Tomography including five Oberwolfach meetings (Applied and Computational Convexity, 1992, 1995, 1999, Proof Verification and Approximative Algorithms, 1994, The Mathematics of Discrete Tomography, 2000) and two Dagstuhl meetings (Counting Issues, 1993, Discrete Tomography: Algorithms and Complexity, 1997). Gritzmann has received major research grants including those from the Alexander von Humboldt-Foundation, the German Science Foundation, the German Ministry of Science and Technology, the Bavarian Ministry for State Development and Ecology, the National Science Foundation, the German Academic Exchange Council and Nato. Also he received funding from various industrial and business companies including Infineon Technology and Deutsche Bank.

12– Tel Aviv team. Most participants of the team have strong and diverse management experience. The team coordinator Milman served during the last two decades as a Head of the Department, the President of the Israel Mathematical Union (2000-2002), the advisor of the Minister of Science, Member of the Board of Directors/Advisory Boards of several mathematical Institutes and of the Government Scientific Academic Press Company (Weizmann Press). He is on the Board of Governors of Tel Aviv University and Ariel College. After the half-year MSRI programme in 1996, which he organised with Ball (team 1), he was on the organising or scientific committees of a couple of dozen conferences/workshops all around the world.

Alon has very broad scientific management experience; he was a Chairman of the School of Mathematics and served on many important university committees. He served on the scientific committees/panels of several International and European Congresses. He is the chairman of the Programme Committee of the next International Congress of Mathematicians, to be held in Madrid in 2006.

Schechtman, Pinkus, Y. Benyamini and Reisner served as chairs and deans of their respective mathematical departments and faculties; Pinkus is presently the President of the Israel Mathematical Union and he is in charge of publications for FoCM (Foundations of Computational Mathematics). Also many other members of this team organised scientific conferences (Krivelevich, Sodin, Schechtman, Gordon, Benyamini, Reisner, Pinkus) and had broad management experience.

The team will host a conference in 2008.

13– Kharkov team. Pastur was the Deputy Director of the Institute for Low Temperature Physics (1987-1997), the Head of the Mathematical Division (same period), Chairman of the Ukrainian Funds for Fundamental Research (1993-1996), and the team coordinator of the Soros and the INTAS grants and several national grants. He is at present the Head of the Department of Theoretical Physics of the Institute. He has also a considerable experience in the scientific organisation of several international conferences and congresses, being a member of the scientific and the panel committees.

Shcherbina is the Head of the Department of Statistical Methods of the Institute for Low Temperature Physics.

Chueshov has experience as Head of the Department of Mathematical Physics and Computational Mathematics at Kharkov National University and of the research group “Long-time dynamics of dissipative systems” at the same department. He is team leader of a 3-year INTAS project.

B5 RELEVANCE TO THE OBJECTIVES OF THE ACTIVITY

Benefits of a Europe-wide project

The nature of this project makes the European Community the ideal organisational level at which to undertake it. On the one hand, no single country possesses the expertise to present a unified picture of all aspects of the theory and its applications; on the other, Europe certainly *does* possess such expertise and is significantly stronger in this respect than the US. The fact that there are over 100 research students currently preparing theses in this network is a strong indication of the vigour of the fields involved.

The study of high dimensional phenomena is just emerging as a well-defined activity, synthesising a number of existing branches of the mathematical sciences and their applications. The scope of its applications is expected to be very wide, including physics (see Section B1.3), biology (neural networks, learning) and social sciences (modelling the stock market, internet, traffic). Awareness of this is just becoming clear to the leading figures in these different branches of mathematics. Now is the perfect time to disseminate this awareness to the younger generation.

The multidisciplinary approach to this area will have an added bonus for the younger researchers involved. Hiring decisions are often strongly influenced by the unfamiliarity of the hiring committees with the candidates' interests. One of the goals of the network is to establish and facilitate contacts between traditionally rather distant domains of the network (geometry and probability, mathematical physics and combinatorics) which will not only bring younger researchers to the attention of many more senior mathematicians but will also acquaint those senior mathematicians with their fields of interest.

Enhancement of training and potential to overcome fragmentation

The overall capacity to train young researchers and transfer knowledge between young and more established scientists will be enormously enhanced by this network. Again, one of the main reasons for this is the considerable breadth of expertise within the network. The recent programme at PIMS in Canada which, despite being located there, was overwhelmingly attended by European mathematicians, brought together around half the members of this network and the benefits were immediately obvious. There were many educational lectures, for example by Alon, on the theory of geometric expanders, by Ball on applications of harmonic analysis to geometry, by Schneider on convex geometry, by Pastur and Soshnikov on random matrices and by McDiarmid on randomised computation. These presented material that is available in the literature but which is dispersed and difficult to locate without the help of an expert. This meeting was one of the reasons that so many leading figures became convinced of the need for a programme such as this one.

The diversity of the subjects belonging to this proposal could be considered to constitute, at present, a fragmentation of the emerging study of high-dimensional phenomena into sub-disciplines. For example, log-Sobolev inequalities have been used for decades within statistical physics but appeared in geometry only 7 or 8 years ago. On the other hand, the highly developed techniques of asymptotic geometric analysis based on the concentration phenomenon, convexity and symmetrisation, have produced a number of powerful inequalities and techniques that will undoubtedly be useful in mathematical physics, for the study of the eigenvalue statistics of random matrices of large size, the thermodynamics of disordered spin systems, but which have been virtually unknown to researchers in those fields. This is a situation which must be rectified and the potential of this programme to do so, is very considerable. As mentioned in earlier sections, the success of programmes such as the one in PIMS and earlier ones in Edinburgh and Berkeley, shows that the network has the capacity to ensure effective communication between the different groups and to avoid fragmentation by sub-discipline.

Long-term collaborations

It is very much to be hoped and there is a strong expectation that this project, and related events such as the semester at IHP mentioned in Section 2.1, will encourage long term collaborations in training and the transfer of knowledge in addition to the many research collaborations outlined in Section 3.3. One reason is that the younger generation of researchers who gradually replace the older, will have seen all mathematical areas of the project presented as part of a whole, instead of picking them up piecemeal over many years. In particular, it is to be expected that combinatorial methods will be incorporated even more into the analytic/geometric areas of the project and that geometric results will improve the efficacy of several of the computational algorithms developed by the combinatorial group. Random matrix methods have already found important uses in combinatorics and geometry.

We expect that the active young scientists who will meet and collaborate during the training and research that takes place within the framework of our programme, will maintain these contacts afterwards, as they become scientific leaders in their universities and come to lead international cooperative research. Those who leave the academic world and enter industry or commerce, will maintain links between themselves and with those who continue to work in science, thereby contributing to advances in technology and the increase of knowledge within society as a whole.

Special need to promote training and knowledge transfer at the EU level

In addition to the benefits outlined at the start of the section there is one overwhelming reason why a pan-European approach is necessary. As explained in Section 2.2, the drain of established scientists and more importantly, students, to the US is a very serious problem for many European countries. Financial considerations are one reason for the drain, but another is the ease with which it is possible in the US to participate in a broad scientific community. The Marie Curie training program, with its emphasis on mobility of young researchers should help to provide similar opportunities in Europe: the reason that this project, especially, is appropriate for such support is that it makes connections between several of the directions in which Europe is strongest. Like many Eastern European countries, Hungary, Poland, and Ukraine have educational systems that have been extremely successful in producing young scientists and especially mathematicians. They continue to produce more than their fair share of exceptionally gifted students but since the collapse of the former Soviet bloc, they are haemorrhaging talent to the US. The fact that this project involves such important mathematical disciplines as probability, combinatorics and mathematical physics in which these countries have historically been strong is critical to the ability of the network to help these countries to retain young people.

The existence of a dynamic network involving some of Europe's most outstanding mathematicians and linking highly active research fields, each with a reputation for the effort they put into the support of young people, will be immensely attractive to younger scientists and will thereby have a considerable effect on increasing the pool of researchers in these areas.

The topics involved in this project lie at the overlap of vital areas of mathematics and mathematical physics: probability theory, geometry, statistical physics and combinatorics. It is clear that the remarkable achievements of the last two decades and the fact that so many of the ideas involved are common to different areas, are not yet sufficiently familiar to all those who stand to benefit from them. The Marie Curie training programme is the perfect vehicle for improving this situation.

B6 ADDED VALUE TO THE COMMUNITY

Towards objectives of the European Research Area
The project will contribute to the main strategic objectives of the European Research Area: strengthening the scientific and technological bases of industry. This will be (indirectly) done in the following two forms. First, increasing the pool of highes level researchers, who are able to works actively in a variety of the Priority Thematic Areas of the FP6. Second, by direct and indirect links of senior researchers with their partners in science, applied science and industry, who are able to spread new promising ideas and methods. Third, by structuring and shaping the emerging interdisciplinary area lying on the intersection of geometry, analysis, probability, combinatorics and mathematical physics and dealing with high dimensional phenomena in this and adjacent field, working out and making explicit new points of view, thereby enlarging European scientific landscape.

Cohesion and integration The associated states represented in this programme are Poland, Hungary and Israel. The principal less-favoured regions which contain nodes are in Spain and Greece.

As mentioned earlier, Poland, Hungary and Israel present no special difficulties in integration because their members already have strong research links with mathematicians in other parts of Europe, within the fields at which they excel. In each case however, these countries have much to gain from the network and much to contribute to it. The bringing together of all these different fields would be impossible without the combinatorial expertise in Hungary, the probabilistic expertise in Poland and the expertise in high-dimensional geometry and combinatorics in Israel. The Hungarian team has identified convex geometry as the area in which it will benefit enormously from contact with other groups, most especially the ones in France, Britain, Italy and Israel: the Polish team lacks expertise in combinatorics which can be found in Britain, Hungary and Israel. The Israeli team has no members working in the area of "Isoperimetric principles" even though they have used such methods extensively.

The teams from less-favoured regions have relatively few highly senior personnel and they will benefit enormously from contact with the distinguished mathematicians elsewhere in the network.

We will also collaborate with colleagues of ours in other European countries (Denmark, Sweden, Norway), and in Russia.

Gender Issues There are two principal aspects to the issue of equal opportunities in science: the small number of women who attracted into the subject to start with, and the question of whether they are treated equally once they are there. The first is well understood to be an extremely serious problem for mathematics. In mathematics, the European Commission's target of 40% participation by women is presently unattainable. The reasons for this are probably rooted in pre-university level education. The proportion of women in this project is around 12% which is above average within mathematics. More importantly, the network does include several female mathematicians of the very highest calibre. It is generally accepted that the presence of such established and visible figures in a field considerably increases its attractiveness to further women, in particular, by providing female scientific advisers for female registered applicants. Network activities of the local teams will encourage female students (before their masters degrees) to pursue a research career and network members will be especially encouraged to offer advice on such careers to predoctoral students.

Concerning the equality question the network appointments committees will scrutinise all appointments and will monitor the proportions of women applying and being appointed. Serious imbalances between these will be carefully investigated.

Increasing the attractiveness of Europe for researchers and increasing European competitiveness In several areas of this project (convex geometry, isoperimetric inequalities, combinatorial phase transitions) Europe is already stronger than its main competitor, the US. While the drain of established scientists to more lucrative positions in the US is well understood and easily visible, a perhaps far more serious problem is the disappearance of talented students who take doctorates in the US and never return. One way to make Europe more attractive to doctoral students is to provide them with the wherewithal to visit other leading researchers in the area of their PhD. Several students in this field from around Europe, already visited UCL under a Marie Curie training site program and these visits were a dramatic success, not only in the quality of the research they completed but, by their own accounts, in terms of the enjoyment and experience that they gained. The obvious implication is that this programme would be exceptionally well received by young people.

The effects of the brain drain (mentioned above) on the mathematical community in Europe are much stronger in Ukraine and other New Independent States. The Kharkov node consists of members of what was one of the world's leading research centres in mathematical physics, geometry and analysis until 1990. Some of the most senior figures left after the fall of the Soviet Union, but several well known mathematicians, including younger ones, remain and talented youngsters continue to engage in thesis programmes. The integration of Kharkov into the network will help the European Community to play a leading role in assisting the mathematical community in Ukraine to develop research at a high international level, in addition to the actions already engaged under the aegis of the INTAS program.

The 6th PCRDT provides very good material conditions for early stage and experienced mathematicians appointed by Research Training networks. The projected network, which includes a large proportion of world leaders in its research area will also provide a very attracting scientific atmosphere. With the possibility opened in the 6th PCRDT to hire up to 30% of early stage and experienced mathematicians from Third countries, part of the recruitment can be done at a world-wide level. The large training and transfer of knowledge program, the high scientific level of the Conferences and Workshops organized by the network, and the possibility to work with top level experts will certainly increase the attractiveness of Europe for researchers working in several parts of the theory of high-dimensional phenomena in mathematics and adjacent sciences, and thus softening the appeal of the financially inspired brain drain overseas. The projected network will also stimulate joint research activity on a very large scale between leading mathematicians, which will increase the research competitiveness of Europe in strategic areas of mathematics. The dozens of students already preparing a thesis in the network nodes will benefit from the training and transfer of knowledge together with early stage and experienced mathematicians to be appointed by the network, and this formation of the new generation of mathematicians needed to replace the many seniors who are approaching or are over 60 will greatly increase in the middle and long term the scientific competitiveness of Europe in the areas covered by the project.

Co-ordination with other activities

This topic is partly mentioned in Section B3.1, in descriptions of the network teams. All teams have their own undergraduate, PhD and postdoc programs in their institutions, with which the program will naturally cooperate. In particular, joint research projects, seminars, and other training and scientific activities will be organized on the local level and will naturally be benefited by the network.

Besides, members of many teams participate in respective national scientific programs: the French research network GDR funded by CNRS, long term programs, funded by the DFG in Germany, analogous programs in Israel, Spain, Ukraine. There are also bilateral programs: for examples the Picasso Programme (the Zaragoza and the Marne-la-Vallée teams), Alliance with London, Procope with Kiel, Polonium with Warsaw, Balaton with Budapest, Arc-en-Ciel with Tel Aviv. Joint German-Israel Programme between Kiel and Tel Aviv, joint Germany-Hungary Programme etc.

David Preiss from the London node recently submitted a renewal application for a Marie Curie training site that he has administered at UCL for the last few years: contract number HPMT-CT-2000-00037. The research topic of this training site overlaps somewhat with a subfield of this proposal.

Last, but not least, the network activity will stimulate cooperation on students mobility in the frameworks of Erasmus-Socrates program existing in many nodes, in Paris, Marne-la-Vallée, Zaragoza, Madrid etc. and will help developing other cooperations. More generally the network will help communication between academics from all over Europe about teaching, a very important issue in the perspective of harmonization of University degrees in Europe (3-5-8) opened at the Bologna meeting.

B7 INDICATIVE FINANCIAL INFORMATION

The computation of the budget depends upon many parameters. To indicate how the budget will work, we take two more or less generic examples. Early-stage researchers are assumed to be single and will be appointed under an employment contract (29000 euros per year). Experienced researchers are assumed to be married and will be appointed under an employment contract (44500 euros per year). As the travel allowance, we took the average amount of 750 euros (1000-1500 km). Transnational mobility will be the rule. By following a calculation from the handbook and the HRM Work Progamme we obtain the first table.

	early-stage researcher salary	experienced researcher salary
Marne-la-Vallée	45451.40	65449.10
London	48275.00	69762.50
Kiel	45125.60	64951.40
Budapest	32528.00	45707.00
Heraklion	39261.20	55992.80
Warsaw	39659.40	56601.10
Zaragoza	41469.40	59366.10
Florence	43496.60	62462.90
Vienna	45198.00	65062.00
Toulouse	45451.40	65449.10
Freiburg	45125.60	64951.40
Tel Aviv	51569.20	74794.80

Estimates of the overall salary expenses for each node for the combination of early-stage and experienced researchers appears in the next table.

Participant	Salary expenses
Marne-la-Vallée	221801,00
London	201193,75
Kiel	66319,90
Budapest	94499,00
Heraklion	67257,60
Warsaw	58045,10
Zaragoza	60785,10
Florence	74728,05
Vienna	55130,00
Toulouse	99537,65
Freiburg	66319,90
Tel Aviv	252728,00
Total	1318345,05

The last table gives an outline budget and shows approximately how expenses will be allocated among the teams. The distribution of funds through the network reflects the sizes of the participants and the location of the node. It also provides for a small allocation to the Kharkov team in order to give some financial autonomy to this group.

Indicative financial information on the network project (excluding expenses related to the recruitment of early-stage and experienced researchers)				
Network Team	Contribution to the research training/ transfer of knowledge expenses (Euro)		Management activities (including audit certification) (Euro)	Other types of expenses/ specific conditions (Euro)
	(A)	(B)	(C)	(D)
1– CNRS	41084, 94	8000	18625, 66	0
2- UCL	48788, 37	2000	19272, 04	0
3– UK	17974, 66	2000	7579, 54	0
4– RI	30813, 70	2000	12451, 42	0
5– UOC	28245, 90	2000	11477, 04	0
6– IM PAN	17974, 66	2000	7579, 54	0
7– UZ	33381, 51	2000	13425, 79	0
8– UNIFI	25678, 09	2000	10502, 67	0
9– TUW	15406, 85	2000	6605, 17	0
10– UPS	28245, 90	2000	11477, 04	0
11– UF	17974, 66	2000	7579, 54	0
12– TAU	35949, 32	2000	14400, 17	0
13– MDILTP	10000	2000	1000	0
Totals	351518,56	32000	141975,62	0

B8 PREVIOUS PROPOSALS AND CONTRACTS

The present proposal is new and is not based on any network already financed by any European programme.

David Preiss from the London node recently submitted a renewal application for a Marie Curie training site that he has administered at UCL for the last few years: contract number HPMT-CT-2000-00037. The research topic of this training site overlaps somewhat with a subfield of this proposal.

B9 OTHER ISSUES

A.

Does the research in this proposal raise sensitive ethical questions related to :	YES	NO
Human beings		X
Human biological samples		X
Personal data (whether identified by names or not)		X
Genetic information		X
Animals		X

B. This project does not involve research activity aimed at human cloning for reproductive purposes; it does not involve research activity intended to modify the genetic heritage of human beings which could make such changes heritable; it does not involve research activity either intended to create human embryos solely for the purpose of research or for the purpose of stem cell procurement, including by means of somatic cell nuclear transfer, and it does not contain any research involving the use of human embryos or embryonic stem cells.

ENDPAGE

HUMAN RESOURCES AND MOBILITY (HRM) ACTIVITY

MARIE CURIE ACTIONS
Research Training Networks (RTNs)

PART B

PHD