Report on the Winter School
"Probabilistic Methods in High Dimension Phenomena"
Toulouse, January 10-14, 2005

The Winter School has gathered 98 registered participants, mostly from Europe. A large part of the attendance was composed of young researchers (predoc or recent postdoc).

The Winter School offered five courses of four hours each, presenting recent developments in the probabilistic analysis of high-dimensional systems:

Random walk and percolation on graphs by Itai Benjamini (Weizmann Institute of Sciences). This course has given an introduction to this topic together with an overview of the recent developments and open problems. The emphasis has been put on the relevance of geometric properties as isoperimetric inequalities and volume growth on graphs. This course belong to the "Asymptotic Combinatorics" and the "High-dimensional phenomena in mathematical physics" parts of the PHD program.

Minkowski sums in Gaussian analysis by Christer Borell (Chalmers Institute of Technology and Göteborg University), presented a new extension of Ehrhard's inequality on the Gaussian measure of sum-sets. Two different approaches were given, together with various applications to the moment problem, or to concavity properties of natural objects in potential theory. This course fully enters the part of the PHD program entitled "Isoperimetric principles in geometry and probability".

Determinantal processes in random matrix theory by Kurt Johansson (K.T.H. Stockholm), provided the participants with an explanation of the occurrence of the Tracy-Widom law in the asymptotic distribution of two apparently unrelated probabilistic objects: the largest eigenvalue of Gaussian Hermitian random matrices, and the longest increasing subsequence in a random permutation. The speaker showed that both phenomena belong to the class of determinantal processes and explained in details the underlying combinatorics. This course belong to the "Asymptotic combinatorics" and the "Mathematical Physics" parts of PHD.

Concentration of functions of independent random variables by Gabor Lugosi (University Pompeu Fabra, Barcelona). This course illustrated the power of the Entropy Method for deriving concentration inequalities. Several applications were provided (e.g. to the bin packing problem, or the fluctuation of the largest eigenvalue of random matrices). The machinery of the method was explained in details, as well as connections to Talagrand's convex distance inequalities. This course is hard to classify as part of a specific area of the PHD project. Indeed concentration of measure is a fundamental tool in most topics covered by PHD as it quantifies the uniformity in large random systems.

Convexity in stochastic geometry by Rolf Schneider (University of Freiburg) showed recent developments in stochastic geometry where natural notions of Brunn-Minkowski theory have been crucial. The first example deals with the calculation of the natural parameters of a Boolean model, providing a natural class of random closed sets. The second example deals with the asymptotic shape of large cells in random mosaics, and the answer to extensions of Kendall's conjecture. This course mixes methods and questions related to several parts of the program as "Isometric convex geometry", 'Isoperimetric Principles in Geometry and Probability".

Special attention has been given to the "diffusion of knowledge": Bibliographic references for the five courses have been posted on the web page of the conference. Moreover, lecture notes of four of the courses were distributed during the conference and posted on the Internet too.

This event was the occasion to provide young and expert researchers with new techniques and results, as well as open problems to work on. It has provided a large audience with five high quality expository courses. It has promoted fruitful exchanges between participants. For all these reasons we are convinced that it will lead to several new mathematical collaborations and discoveries.